Detecting a preformed pair phase: The response to a pairing forcing field

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New Generation in Strongly Correlated Electron Systems

Trogir, September 14, 2015

Outline

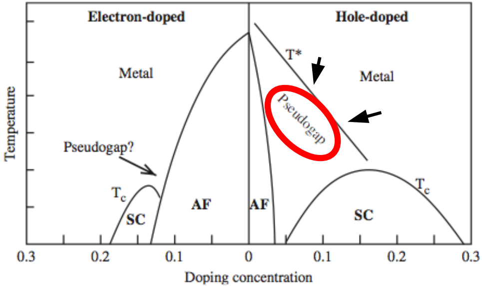
Introduction: Physics of the Pseudogap

Cuprates phase diagram and the pseudogap investigation

Pseudogap physics in high-T_c superconductors

High-T_c superconductors

J. G. Bednorz and K. A. Müller, *Z. Phys. B Condens. Matter* (1986)



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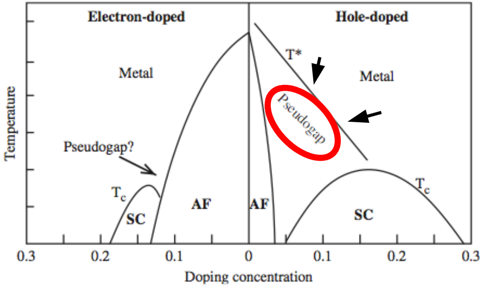

Main features of the **Pseudogap**:

- **Suppression** of spectral weight at Fermi level → observed in spectra (ARPES, optics..) and thermodynamics (NMR)
- charge and spin pattern, but **no** clear signatures of long range **order**

Experimental evidences

Several techniques available

NMR ARPES STM ...

Pump-probe spectroscopy

[A. Cavalleri *Phys. Rev. B* (2014), F. Cilento *Nat. Commun.* (2014)]

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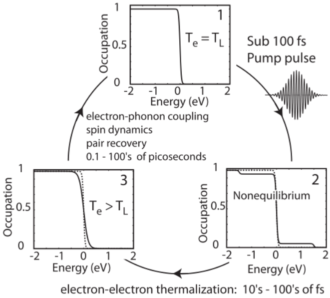


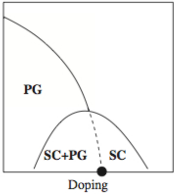
Manipulation of the Pseudogap
to trigger new thermodynamic phase
PG $\Rightarrow$ SC (or a more conducting metal?)

Theoretical models

Debate on the pseudogap physics

Competing-order scenario

- Weak-coupling theories
  [D. J. Scalapino *Rev. Mod. Phys.* (2012)]
- Spin Fluctuations
  [B. Kyung *Phys. Rev. B* (2006)]
- ...



Preformed-pair scenario $\rightarrow |\Delta| \neq 0 \ \langle\Delta\rangle = 0$

- RVB
  [P.W. Anderson, *Science* (1987)]
- BCS-BEC crossover physics
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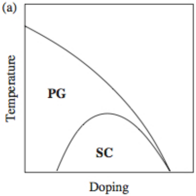

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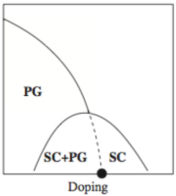

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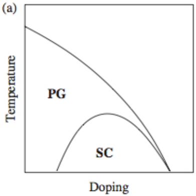


Aim of the work   Set a solid test-bed to detect the presence of **preformed pairs**

Tools

Hubbard model and dynamical mean field theory

Our basic model

The Hubbard model

Hubbard model:

$$H = \underbrace{-t\sum_{\langle ij\rangle\sigma}(c^{\dagger}_{i\sigma}c_{j\sigma}+h.c)}_{H_{\mathbf{kin}}}\underbrace{+U\sum_{i}\left(n_{i\uparrow}-\frac{1}{2}\right)\left(n_{i\downarrow}-\frac{1}{2}\right)}_{H_{\mathbf{U}}} \qquad \Rightarrow \text{Half-filling}$$

Attractive (U<0)		Repulsive (U>0)
↓	**Mapping** $\Longleftrightarrow$	↓
s-wave **SC** + **CDW**		**AFM**

Our basic model

Test-bed model

..Response to a **pairing forcing field** η:

$$H = \underbrace{-t \sum_{\langle ij \rangle \sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + h.c)}_{H_{\mathbf{kin}}} \underbrace{+ U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right)}_{H_{\mathbf{U}}} \underbrace{- \eta \sum_{i} (c_{i\downarrow} c_{i\uparrow} + \mathrm{h.c})}_{H_{\eta}}$$

Attractive (U<0) Repulsive (U>0)

↓ ↓

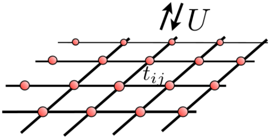
s-wave **SC** + **CDW** $\overset{\textbf{Mapping}}{\Longleftrightarrow}$ **AFM**

Paring coupling

Preformed pairs **couple** with η

Magnetic moments **do not couple** with η

Tool: Dynamical Mean-Field Theory

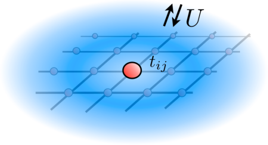


Exact solutions only available

in **one** and **infinite** dimensions
($d \to \infty$)

See previous lecture by Jan Tomczak

Tool: Dynamical Mean-Field Theory



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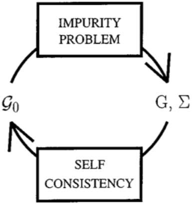
Approximantion:

Looking at the lattice
in a **MF**-fashion
by an effective bath

MFT $\rightarrow$ **No** correlations

DMFT $\rightarrow$ **No** spatial correlations
Yes temporal correlations



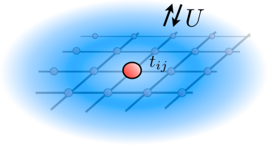
Self-Consistency

$$\mathcal{G}_0(i\omega_n)^{-1} = i\omega_n + \mu + G(i\omega_n)^{-1} - R[G(i\omega_n)]$$

$$G(\tau-\tau') = -\langle Tc(\tau)c^\dagger(\tau')\rangle_{S_{eff}} \qquad \Sigma(i\omega_n) = \mathcal{G}_0(i\omega_n)^{-1} - G(i\omega_n)^{-1}$$

Auxiliary Problem: **Anderson Impurity Model (AIM)**

Tool: Dynamical Mean-Field Theory



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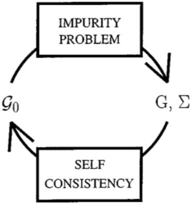


Broken Symmetry:

Matrix in the Nabu formalism
Self-Consistency

$$\widehat{\mathcal{G}}_0(i\omega_n)^{-1} = i\omega_n\hat{\tau}_0 + \mu\hat{\tau}_3 - t^2\hat{\tau}_3\widehat{G}(i\omega_n)\hat{\tau}_3$$

Bethe Lattice

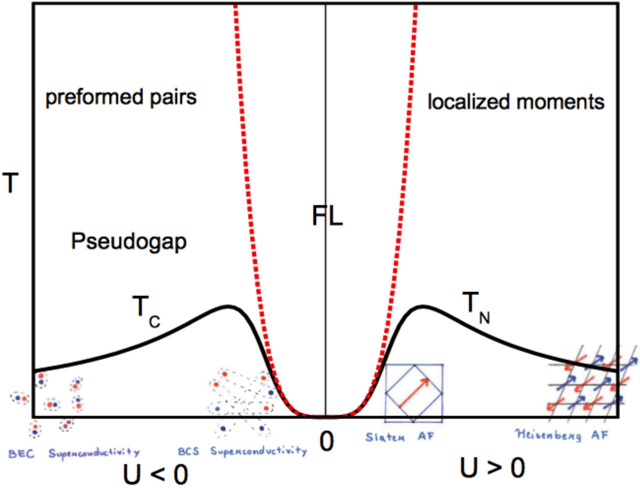
Auxiliary Problem: **Superconducting Anderson Impurity Model (AIM)**

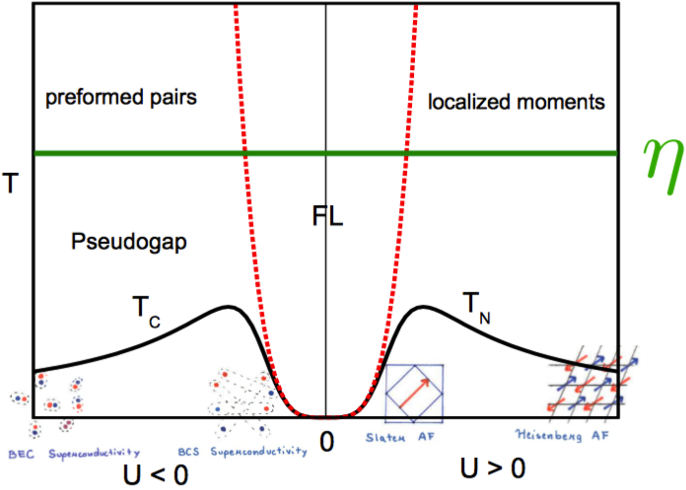
Work and Results

DMFT results under a pairing forcing field: the superconducting response

Detecting preformed pairs

Attractive and Repulsive Hubbard model: the phase diagram

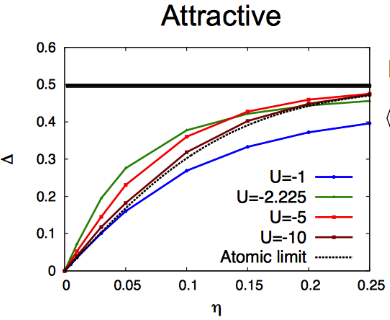




half filling

DMFT results under a forcing pairing field

$$H = H_{\text{kin}} + H_U \underbrace{- \eta \sum_i (c_{i\downarrow} c_{i\uparrow} + \text{h.c})}_{H_\eta} \quad \Rightarrow \Delta(\eta) = \frac{1}{N} \sum_i \langle c_{i\downarrow} c_{i\uparrow} \rangle$$



Regime Value

$$\langle c_{i\downarrow} c_{i\uparrow} \rangle \xrightarrow[\eta \to \infty]{} \frac{1}{2}$$

$$\beta = \frac{1}{k_B T} = 7D^{-1}$$

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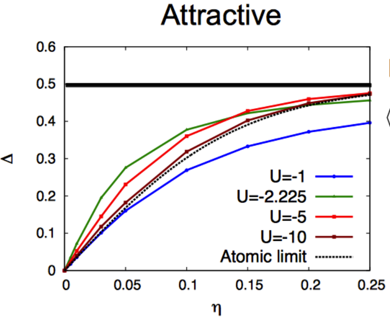

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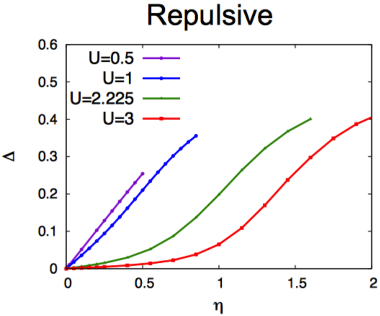

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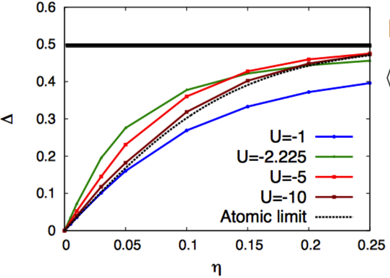
Attractive



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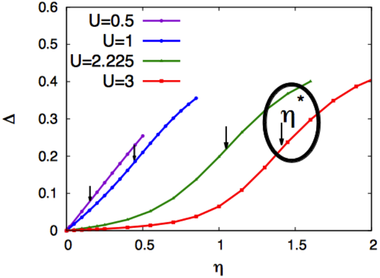
$$\langle c_{i\downarrow} c_{i\uparrow} \rangle \xrightarrow[\eta \to \infty]{} \frac{1}{2}$$

Repulsive



$$\frac{\partial^2 \Delta(\eta)}{\partial \eta^2} < 0$$

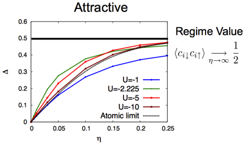
$$\frac{\partial^2 \Delta(\eta)}{\partial \eta^2} > 0 \qquad \eta \in [0, \eta^*)$$

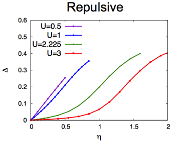
$$\beta = \frac{1}{k_B T} = 7D^{-1}$$

Interpretation

Highlights of the results

- **Suppression** of the linear response for $U > 0$
- **Positive** second derivative $\not\rightleftarrows$ **absence** of preformed pairs





Physical interpretation

$$\Delta(\eta) = \Delta(\bar{\eta}) + \frac{\partial \Delta}{\partial \eta}\bigg|_{\bar{\eta}} (\eta - \bar{\eta}) + \frac{\partial^2 \Delta}{\partial \eta^2}\bigg|_{\bar{\eta}} (\eta - \bar{\eta})^2 + ..$$

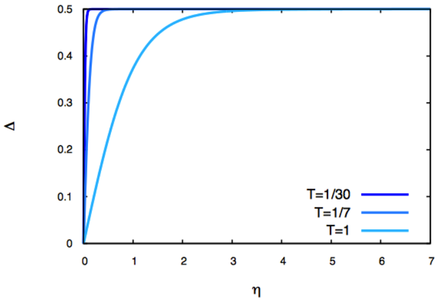
- $\Delta(\bar{\eta})\Big|_{\bar{\eta}=0} = 0$ above the critical temperature;
- $\chi_\Delta = \frac{\partial \Delta}{\partial \eta}\Big|_{\bar{\eta}=0} \rightarrow$ Fluctuation-Dissipation theorem
- $\frac{\partial^2 \Delta}{\partial \eta^2}\Big|_{\bar{\eta}} \rightarrow$ physical meaning?

Why is the second derivative sensitive to preformed pairs?

Atomic limit

Attractive $U < 0$

$$\Delta_{att}^{hf}(\eta) = \frac{1}{2}\,\frac{\sinh(\beta\eta)}{\cosh(\beta\eta) + e^{-\beta\frac{|U|}{2}}}$$



Repulsive $U > 0$

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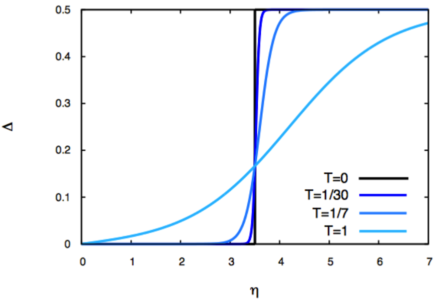

$n_s = 5$ $\quad |U| = 7D$

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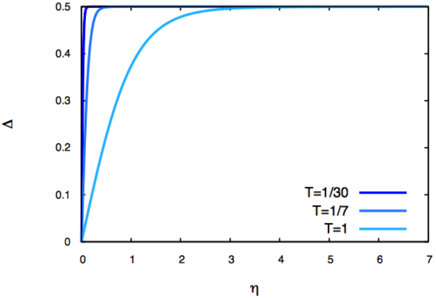

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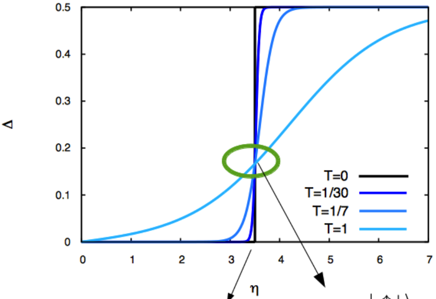


$$\eta = \frac{U}{2}$$

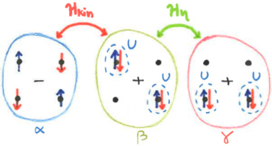
$$|\uparrow\rangle \rightarrow \frac{|\uparrow\downarrow\rangle - |0\rangle}{\sqrt{2}}$$

$n_s = 5$ $|U| = 7D$

Why is the second derivative sensitive to preformed pairs?

Two-sites model: ground state

$$H = H_{\text{kin}} + H_{\text{U}} + H_{\eta}$$



Ground State Crossover

$$|\text{GS}\rangle = \underbrace{\alpha\Big(\frac{|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle}{\sqrt{2}}\Big)}_{\text{localized momenta}} + \underbrace{\beta\Big(\frac{|\uparrow\downarrow,0\rangle + |0,\uparrow\downarrow\rangle}{\sqrt{2}}\Big) + \gamma\Big(\frac{|0,0\rangle + |\uparrow\downarrow,\uparrow\downarrow\rangle}{\sqrt{2}}\Big)}_{\text{localized preformed pairs}}$$

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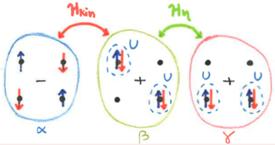

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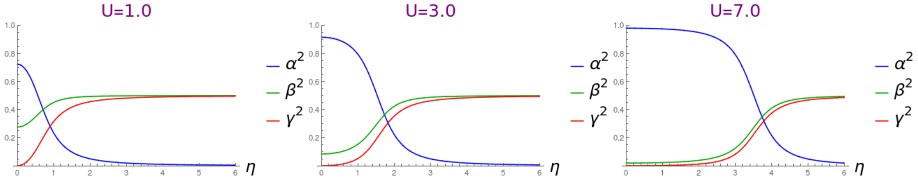

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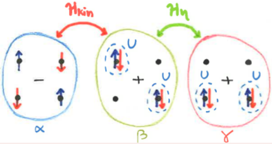

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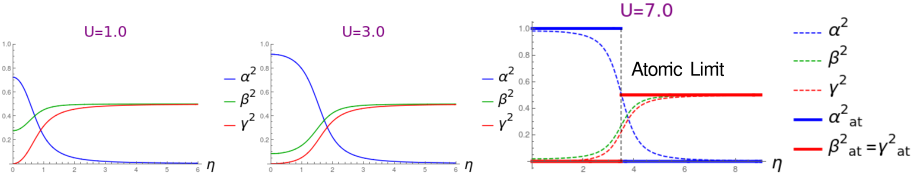

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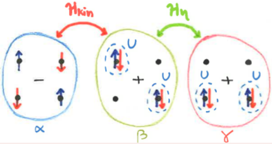

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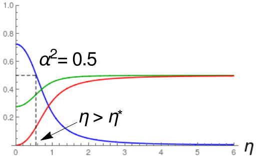
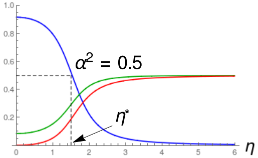
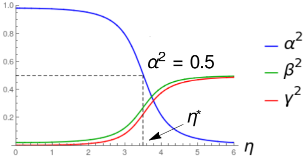
U=1.0



U=3.0



U=7.0



Physical interpretation of η^* $\Rightarrow$ change of the main character of the GS

Interpretation

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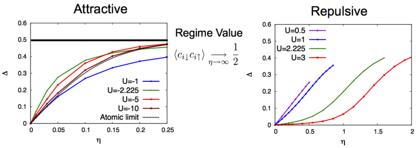
- **Positive** second derivative $\overset{\Rightarrow}{\not\Leftarrow}$ **absence** of preformed pairs

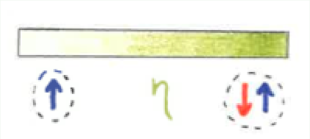


Physical interpretation

Linear response → Fluctuation-Dissipation theorem

Second derivative $\frac{\partial^2 \Delta(\eta)}{\partial \eta^2}$ → direct physical meaning?

Main **GS character** crossover: AFM → SC



Testing preformed pairs in pseudogap region

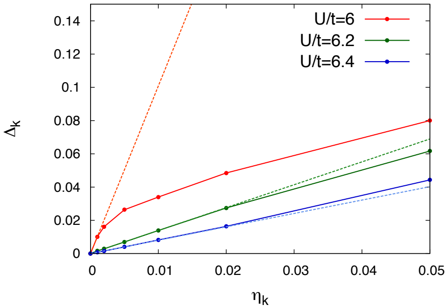
E. Gull and A. J. Millis *Phys. Rev. B* **86** (2012)

DCA calculations in a repulsive Hubbard model under a d-**wave** external pairing field

$\Rightarrow$ Inclusion of short range spatial correlations

$\Rightarrow$ Explicit d-wave superconducting symmetry breaking

$$\Delta_{\mathbf{k}}(\eta_{\mathbf{k}})|_{\mathbf{k}=(0,\pi)}$$



Abrupt response suppression for $U > 5.9t$

Solid interpretation of the results beyond the linear response

$$\frac{\partial^2 \Delta}{\partial \eta^2}|_{U=6.4t} > 0$$

No **preformed pairs**

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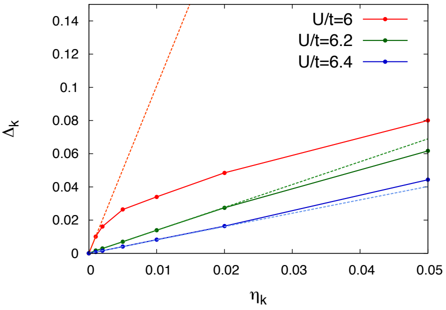
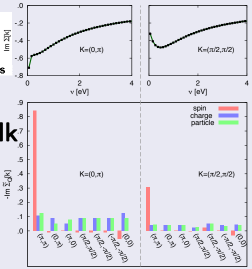


See other **works**:

- J. Merino and O. Gunnarsson *PRB* (2014)
- O. Gunnarsson, **T. Schäfer**, J. P. LeBlanc, E. Gull, J. Merino, G. Sangiovanni, G. Rohringer, and A. Toschi *PRL* (2015)

Fluctuation Diagnostics

See next talk



Conclusions and Outlook

Conclusions and Outlook

Results

DMFT studies under a pairing forcing field η: **Attractive** vs **Repulsive** Hubbard model

- **Second order** response
  - $U < 0 \Rightarrow \frac{\partial^2 \Delta(\eta)}{\partial \eta^2} < 0$ (preformed pairs)
  - $U > 0 \Rightarrow \frac{\partial^2 \Delta(\eta)}{\partial \eta^2} > 0$ for $\eta \in [0, \eta^*)$ (no preformed pairs)
- Physical understanding: **two-sites model**

$$\Delta(\eta) \text{ inflection point} \Leftrightarrow \text{GS main character change}$$

- Improving interpretation of DCA results
  $\frac{\partial^2 \Delta_{\mathbf{k}}}{\partial \eta_{\mathbf{k}}^2}|_{\mathbf{k}=(0,\pi)} > 0 \Rightarrow$ **No preformed pairs**

Outlook

- Probe different ordered phases
- Detect localized magnetic moments with a physical field
- Connection with **out-of-equilibrium** experiments

Thank you for the attention!