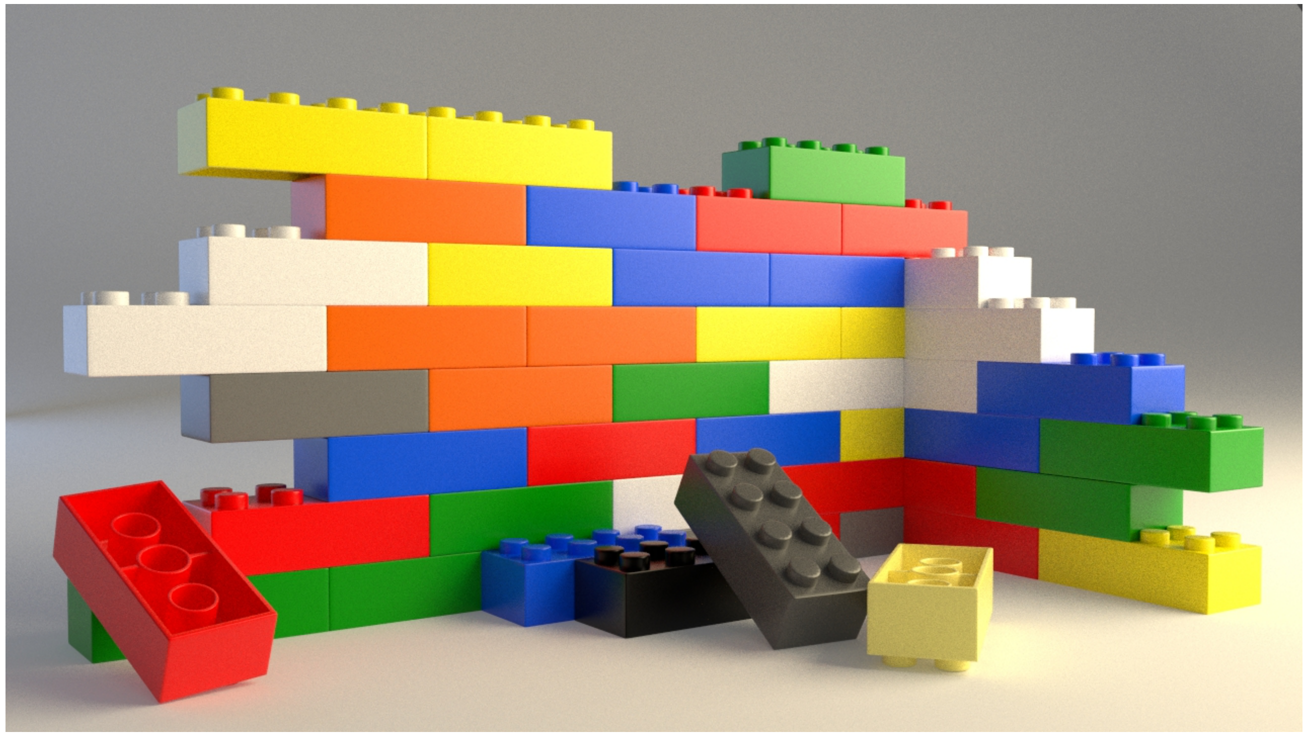
Quantum LEGO:

from quantum wires to 3D topological phases

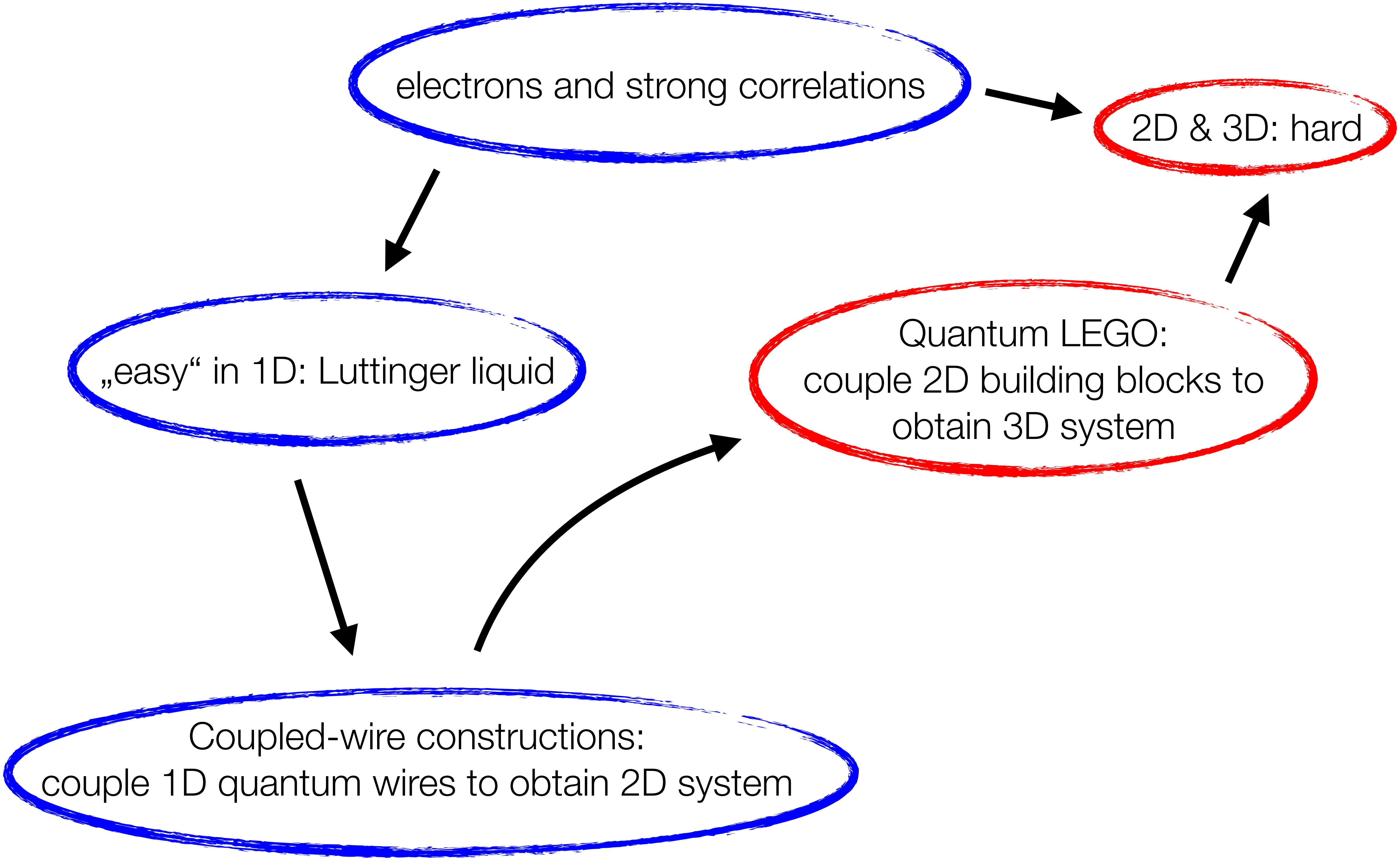


Tobias Meng (Dresden University)

arXiv:1506.01364 + work in progress

(similar: E. Sagi and Y. Oreg, arXiv:1506.02033)

Overview



electrons and strong correlations

2D & 3D: hard

„easy“ in 1D: Luttinger liquid

Quantum LEGO:
couple 2D building blocks to
obtain 3D system

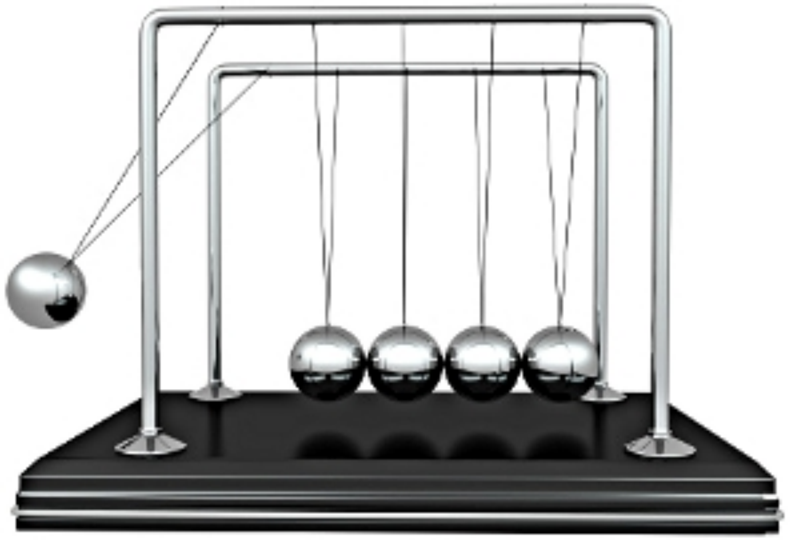
Coupled-wire constructions:
couple 1D quantum wires to obtain 2D system

Solving an interacting 1D system: Luttinger liquid

- In 1D: interacting electrons form „Luttinger liquid“, not Fermi liquid

  $\longrightarrow$ low energy excitations: density waves = bosons

  $\longrightarrow$ density-density interactions can be treated exactly



density: $\rho(x) = -\frac{1}{\pi}\partial_x\phi(x)$

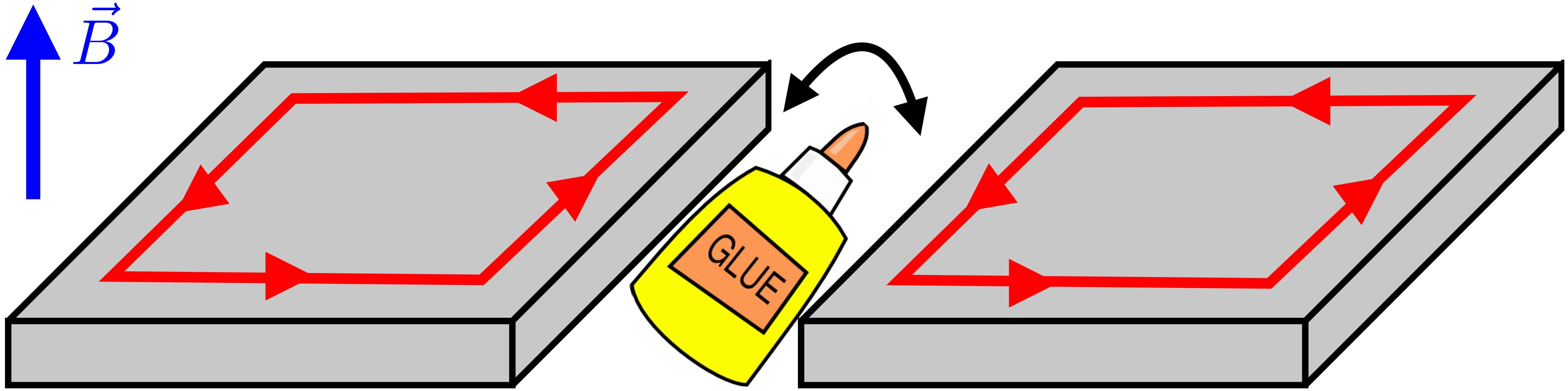
current: $j(x) = \frac{1}{\pi}\partial_x\theta(x)$

$[\phi(x),\theta(x')] = i\frac{\pi}{2}\,\mathrm{sign}(\mathrm{x}'-\mathrm{x})$

fermions: $\Psi_{R,L} \sim e^{i(\pm\phi-\theta)}$

Coupled-wire constructions: going from 1D to 2D

- Integer quantum Hall state & its edge mode:

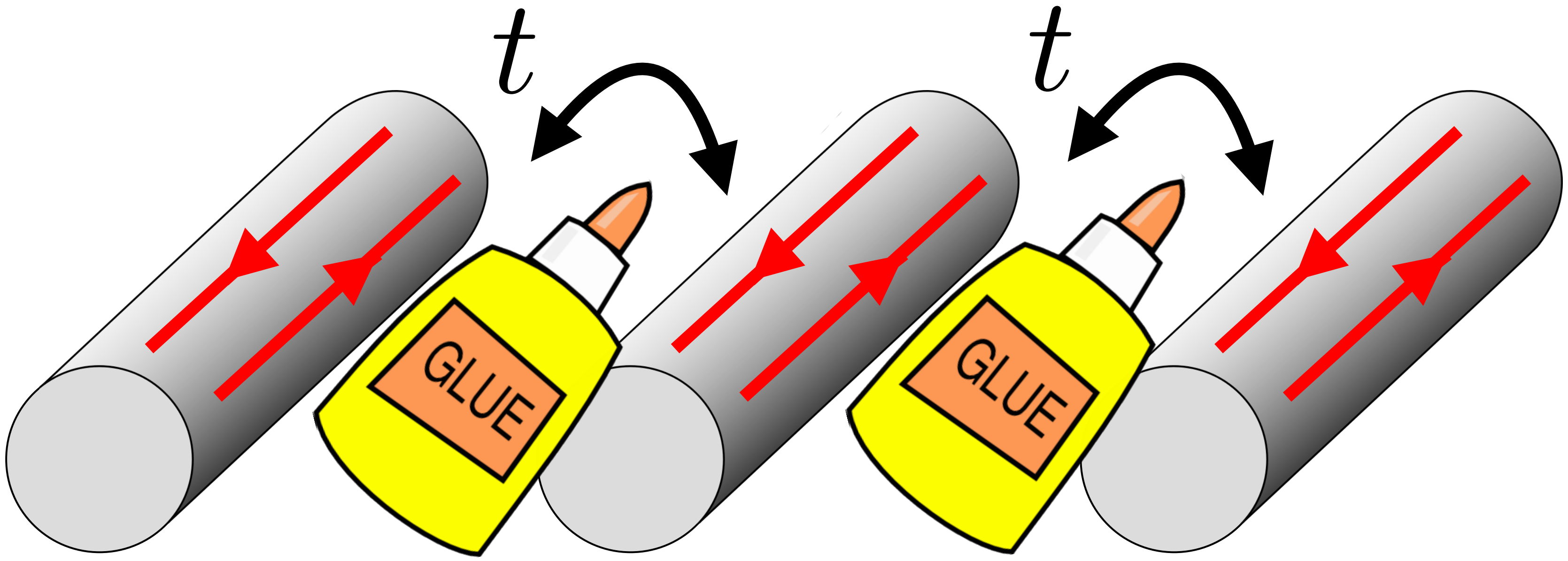


Coupled-wire constructions: going from 1D to 2D

Incomplete list of references:

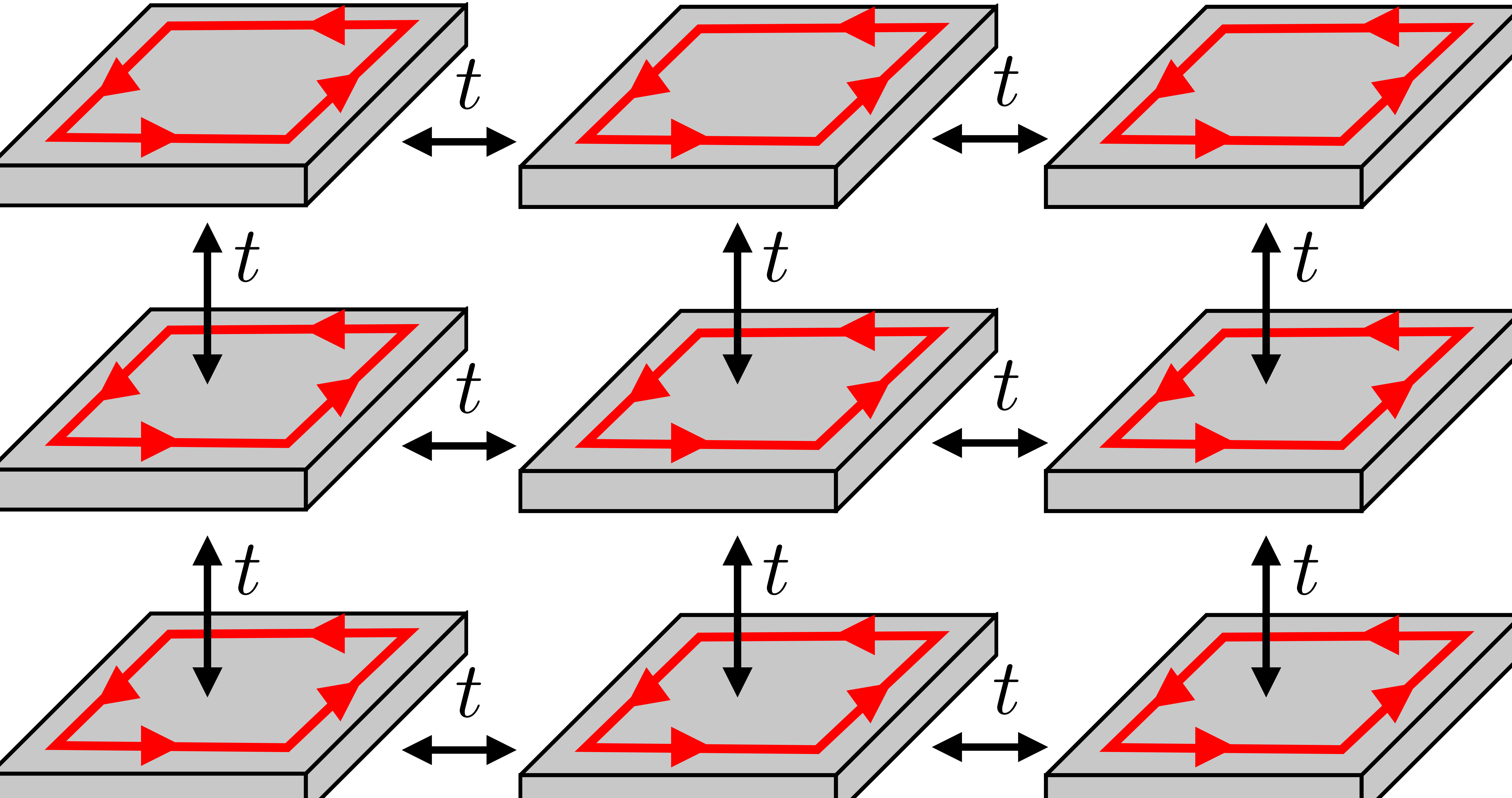
- Abelian FQHE (Laughlin states): Kane, Mukhopadhyay, Lubensky, PRL **88**, 036401 (2002)
- Non-Abelian FQHE: Teo, Kane, PRB **89**, 085101 (2014), arXiv in 2011

- Laughlin-like Abelian topological insulators: Klinovaja, Tserkovnyak, PRB **90**, 115426 (2014)
- Non-Abelian topological insulators: Sagi, Oreg, PRB **90**, 201102(R) (2014)
- Symmetry-based classification: Neupert, Chamon, Mudry, Thomale, PRB **90**, 205101 (2014)
- Chern-Simons description: Santos, Huang, Gefen, Gutman, PRB **91**, 205141 (2015)
- Non-Abelian quantum spin liquids: TM, Neupert, Greiter, Thomale, PRB **91**, 241106(R) (2014)
- Spontaneous TRS symmetry breaking in fractional TIs: TM, Sela, PRB **90**, 235425 (2014)

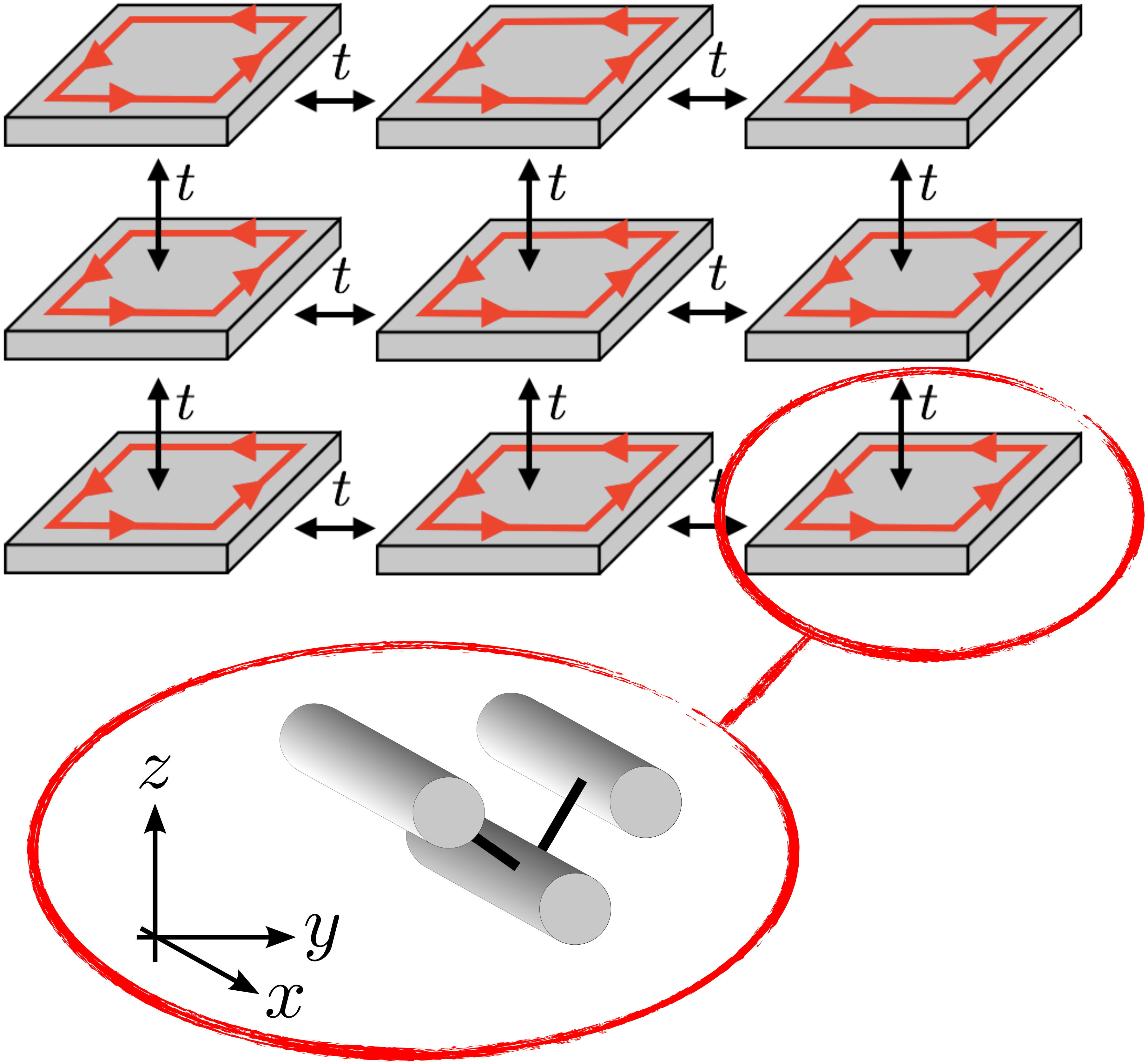


Quantum LEGO in 3D

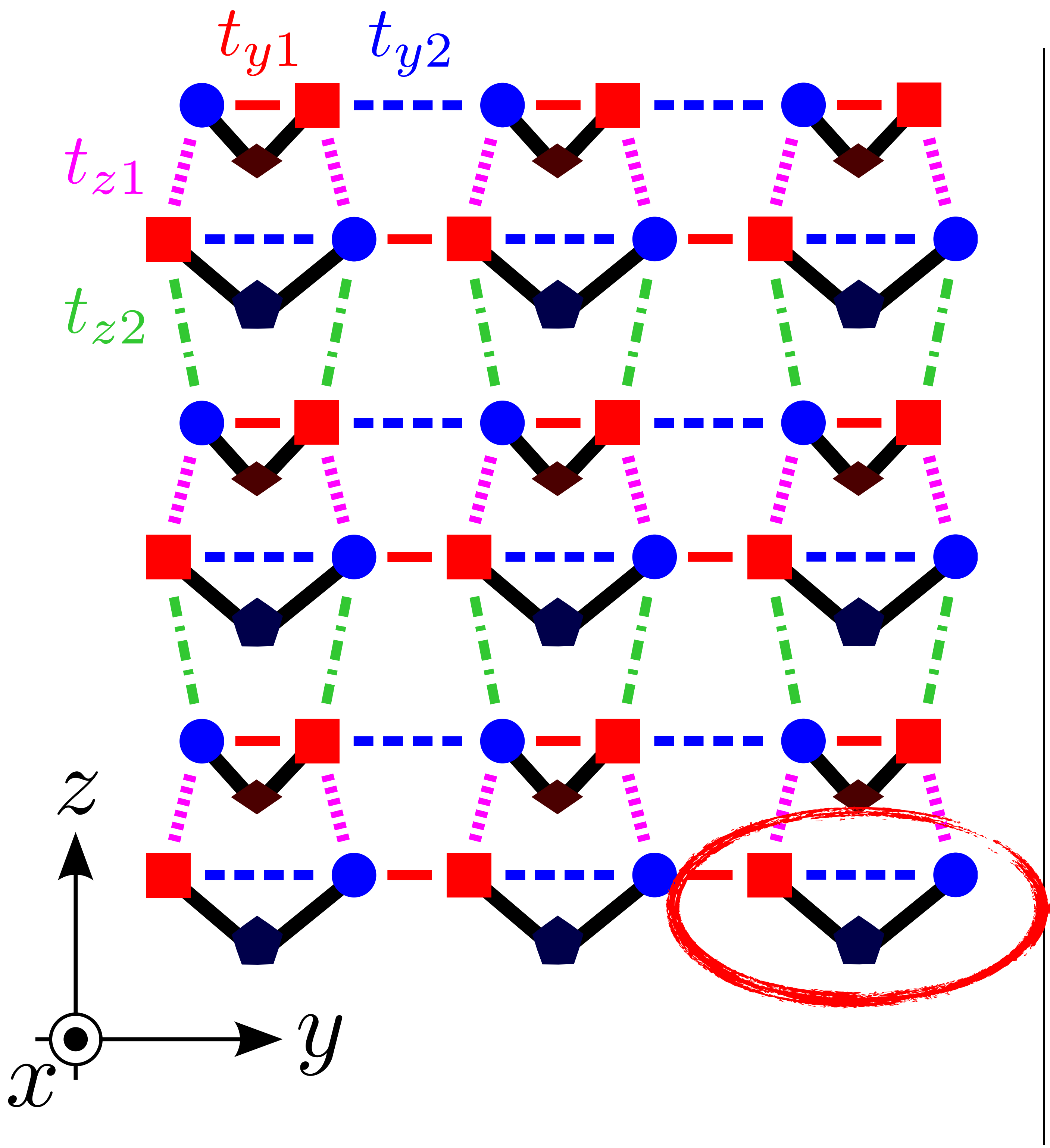
- Constructing topological 3D systems:



Integer quantum Hall blocks (no electron-electron interactions)



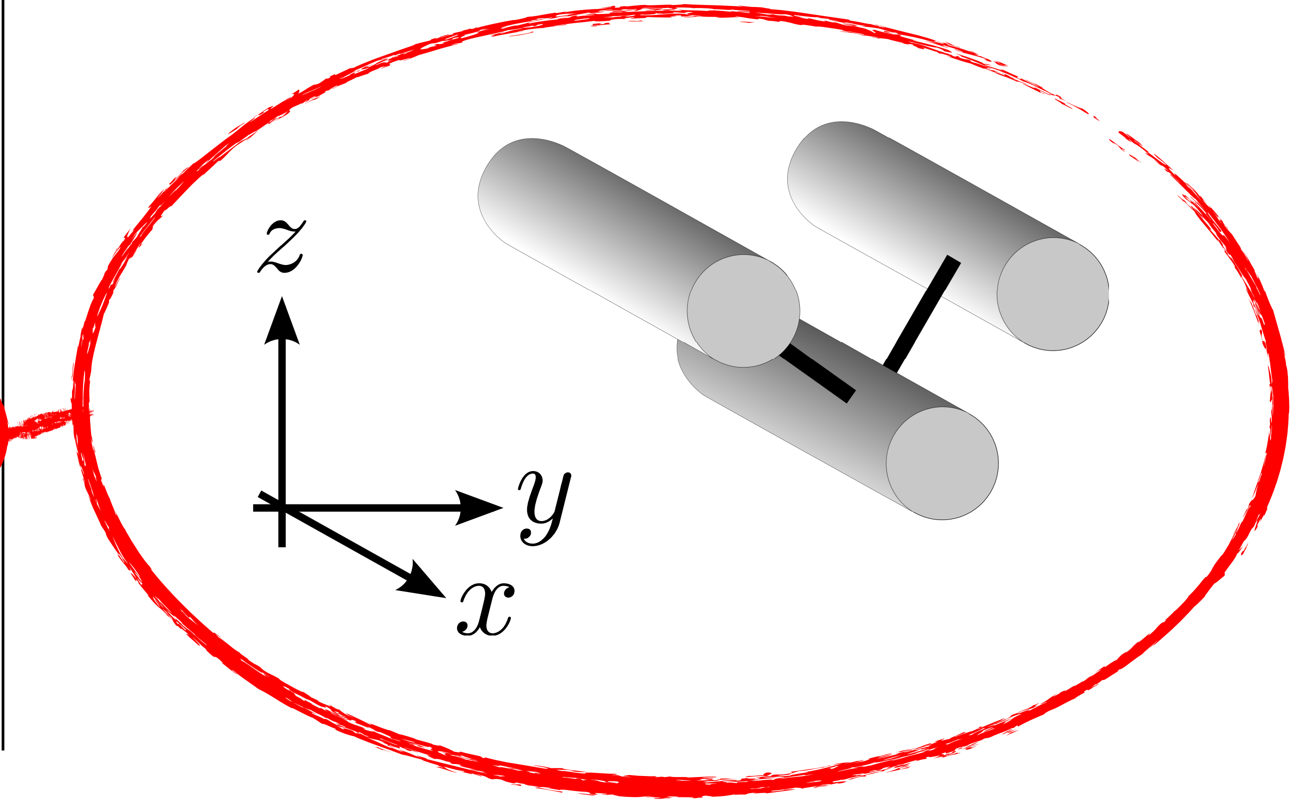
3D system as array of building blocks



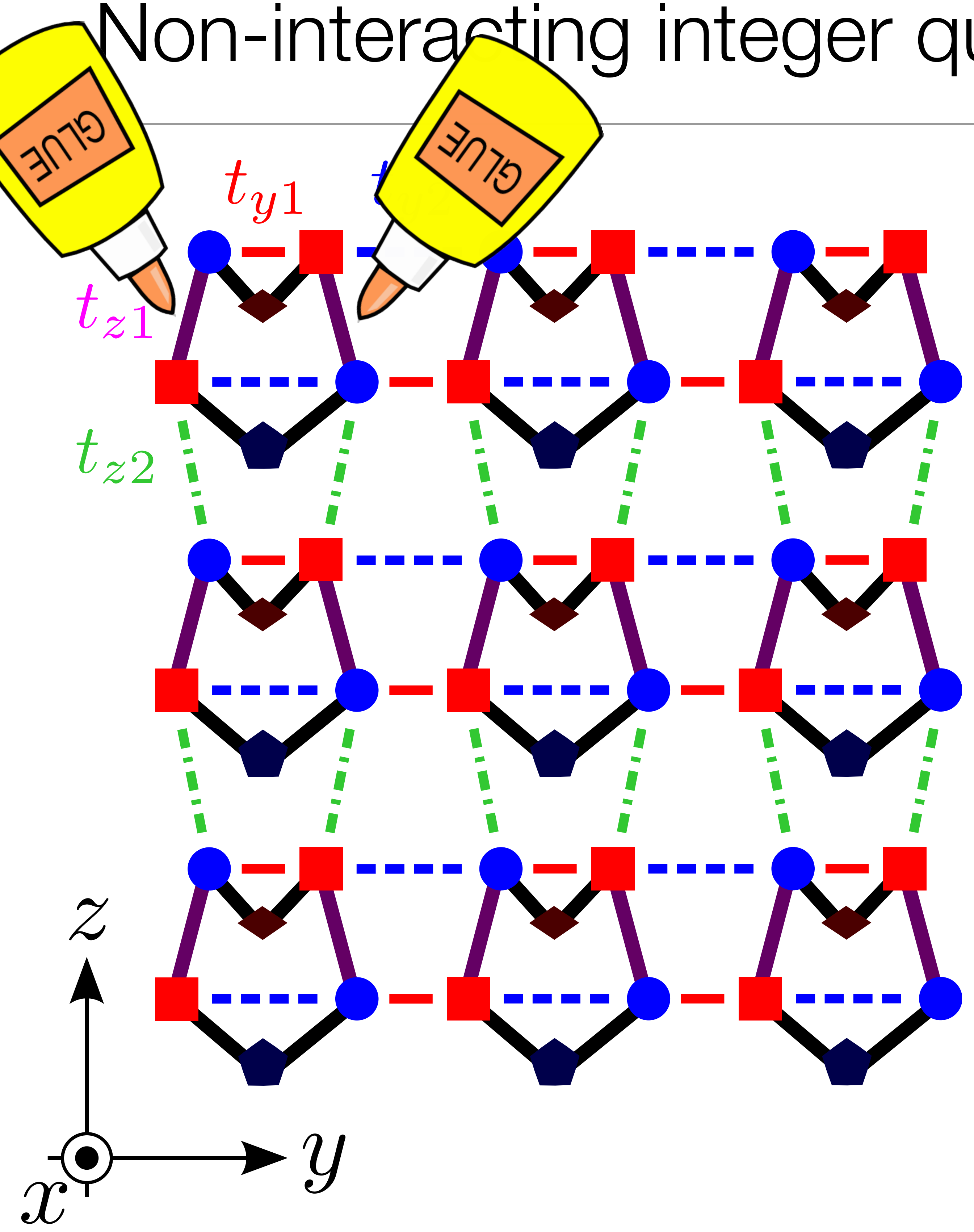
🔵 = right-moving edge mode

🟥 = left-moving edge mode

━ = strong hopping (bulk)

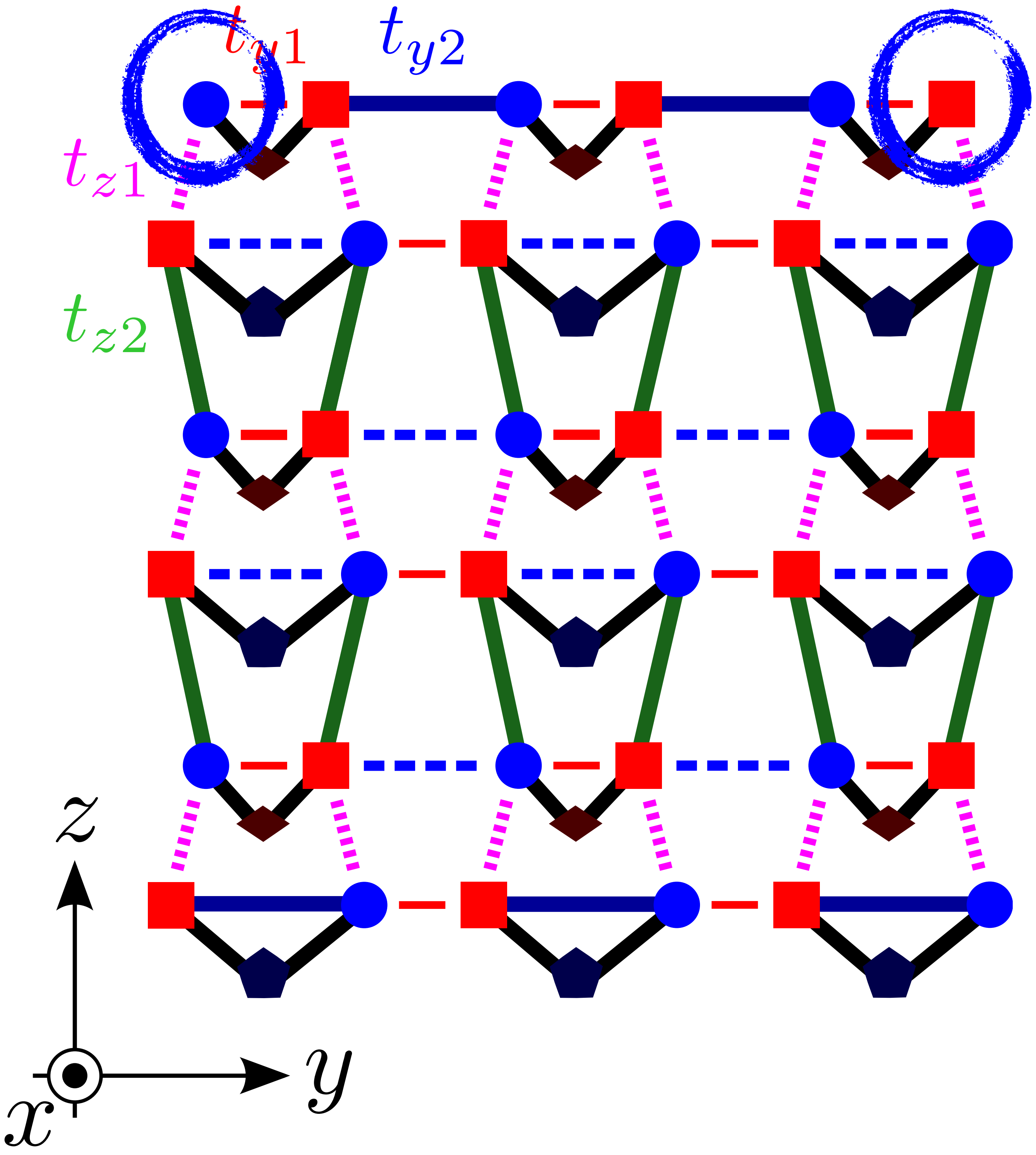


Non-interacting integer quantum Hall blocks: limits



t_{z1} strongest: Normal Insulator (NI)

Non-interacting integer quantum Hall blocks: limits

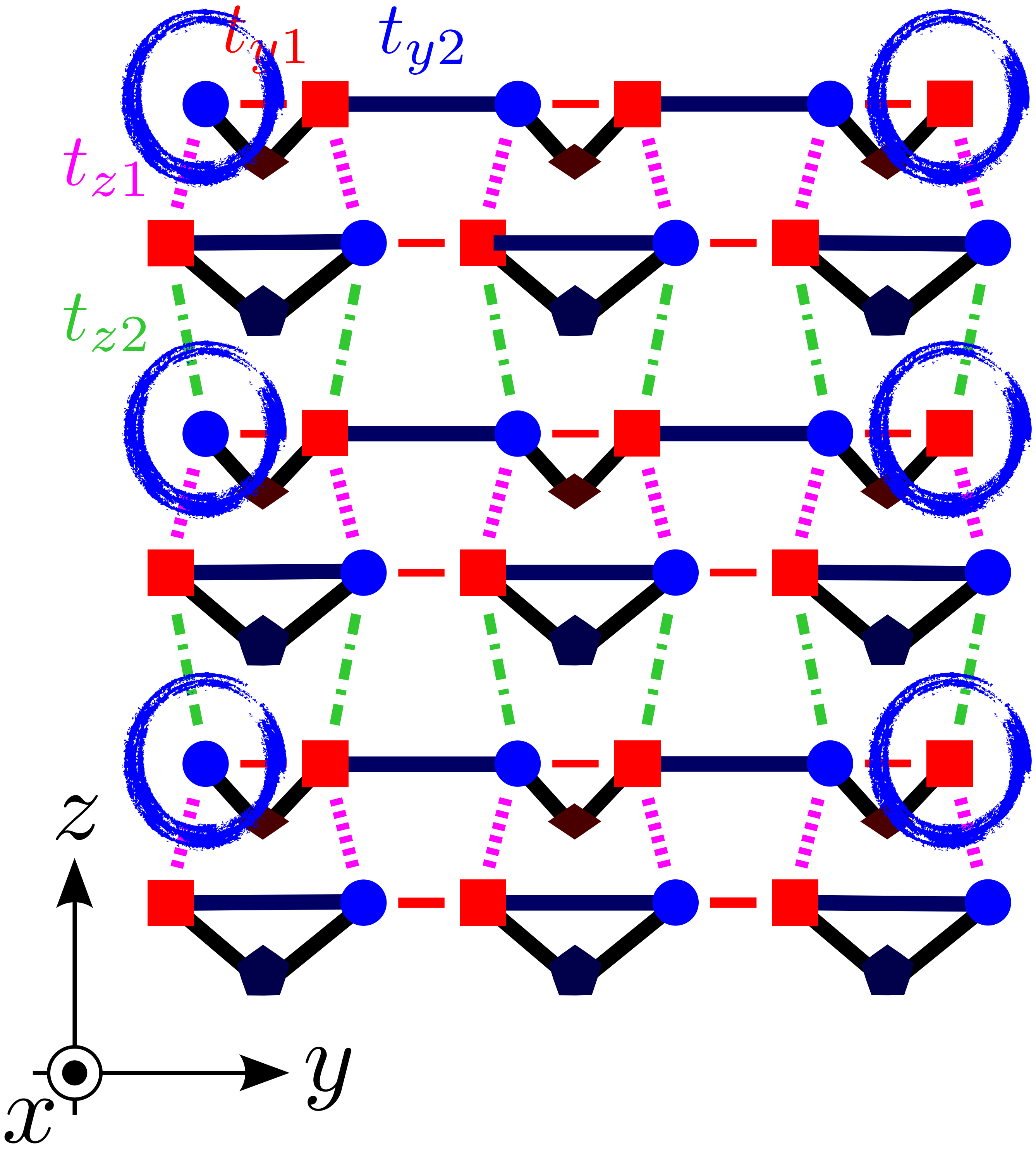


t_{z1} strongest: Normal Insulator (NI)

t_{z2} strongest, $t_{y2} > t_{y1}$:

Single Surface Quantum Anomalous Hall (SSQAH)

Non-interacting integer quantum Hall blocks: limits



t_{z1} strongest: Normal Insulator (NI)

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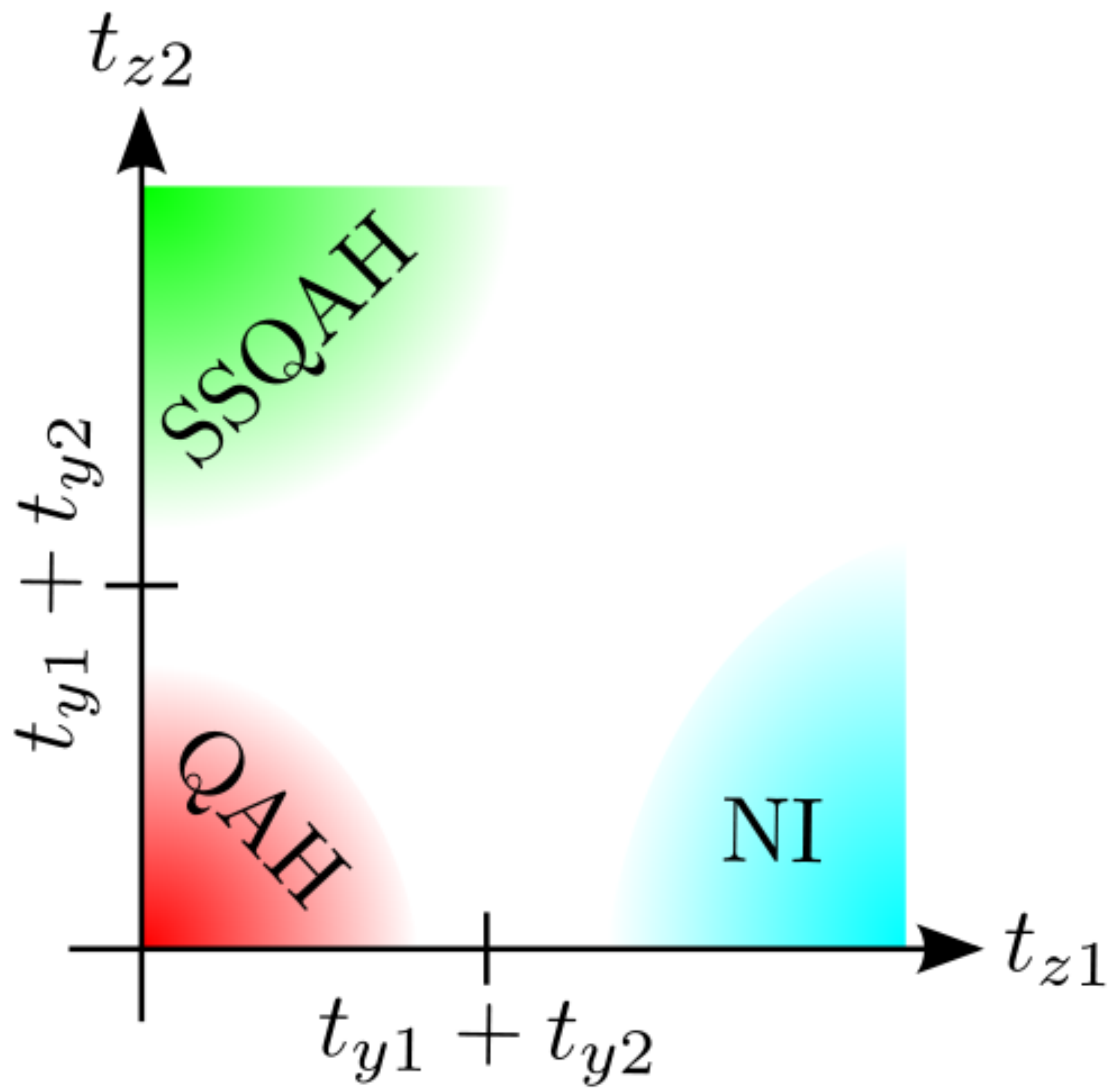
t_{y2} strongest: Quantum Anomalous Hall (QAH)

Non-interacting integer quantum Hall blocks: limits



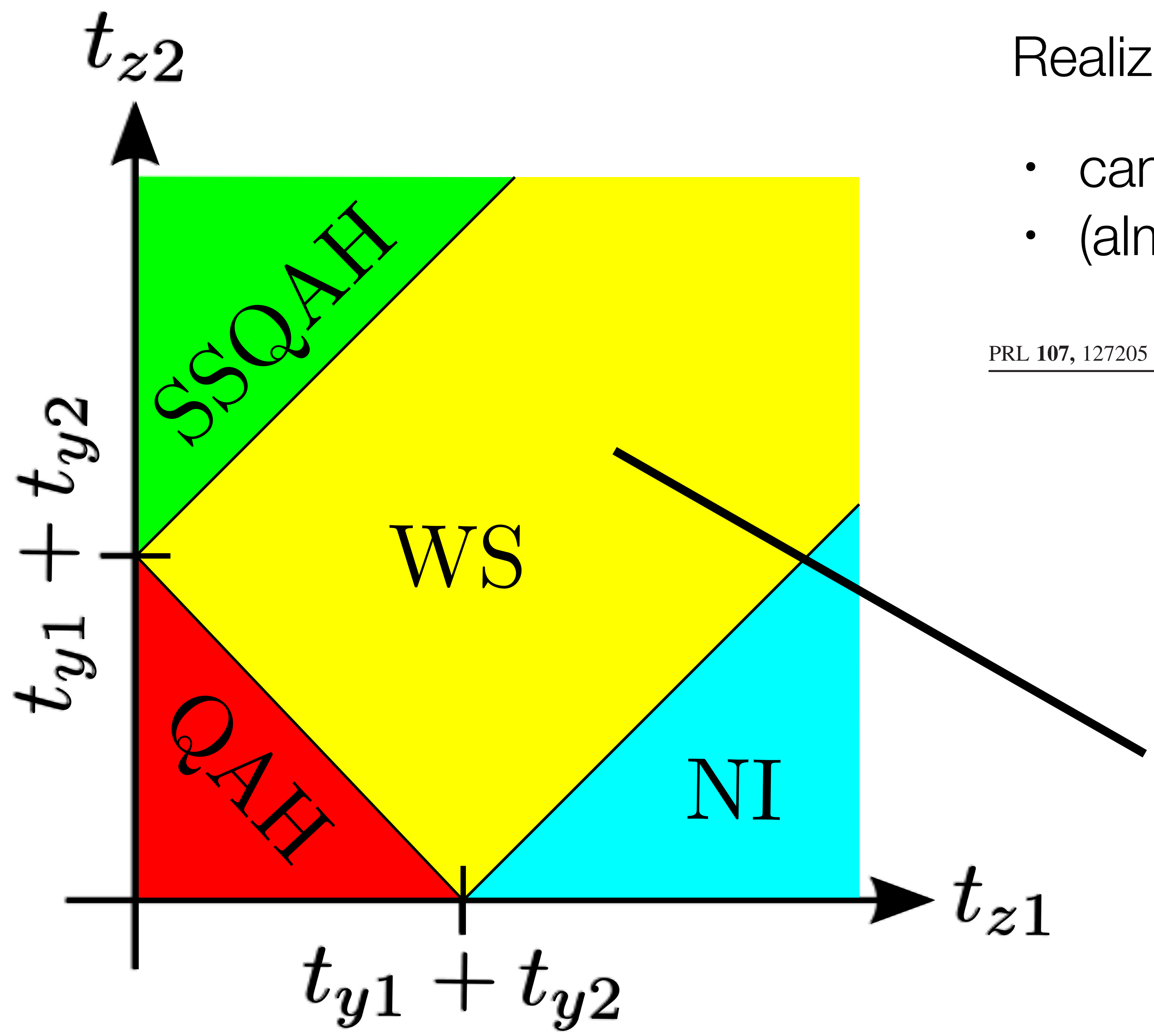
t_{z1} strongest: Normal Insulator (NI)

t_{z2} strongest, $t_{y2} > t_{y1}$:

Single Surface Quantum Anomalous Hall (SSQAH)

t_{y2} strongest: Quantum Anomalous Hall (QAH)

Non-interacting integer quantum Hall blocks



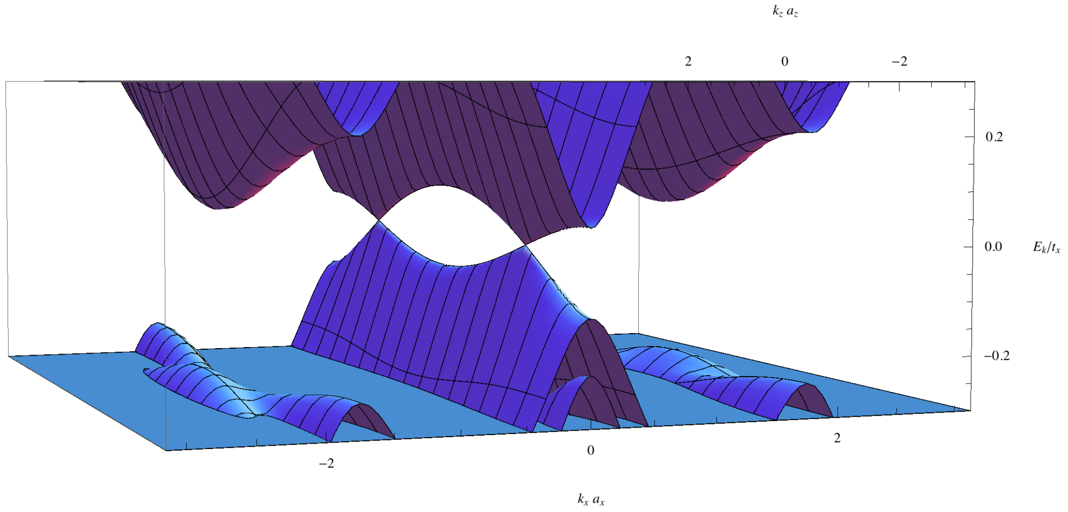
Realization: non-interacting fermionic model

- can be solved exactly
- (almost) same low-energy theory as

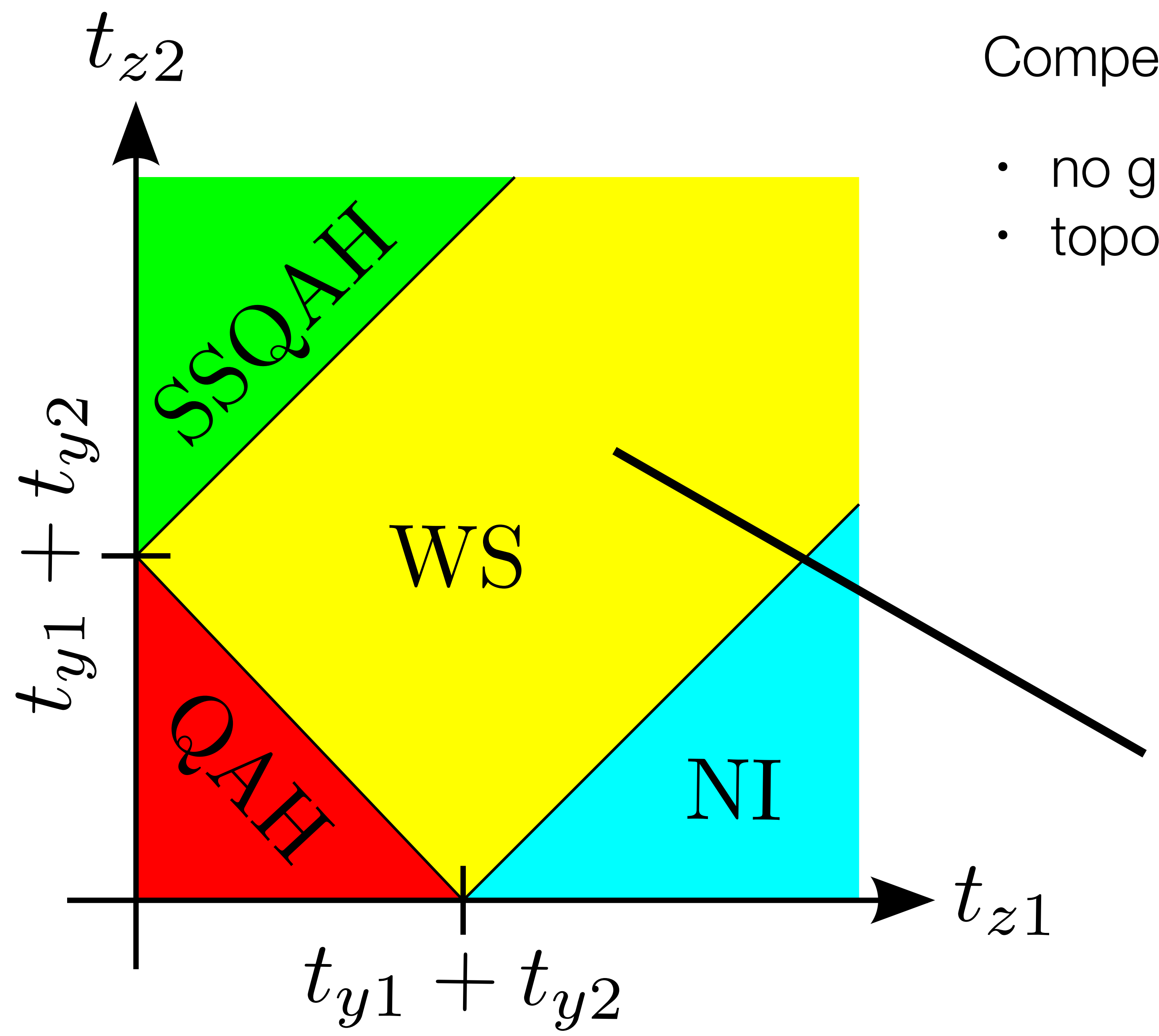
PRL **107,** 127205 (2011) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2011

Weyl Semimetal in a Topological Insulator Multilayer

A. A. Burkov[1,2] and Leon Balents[2]

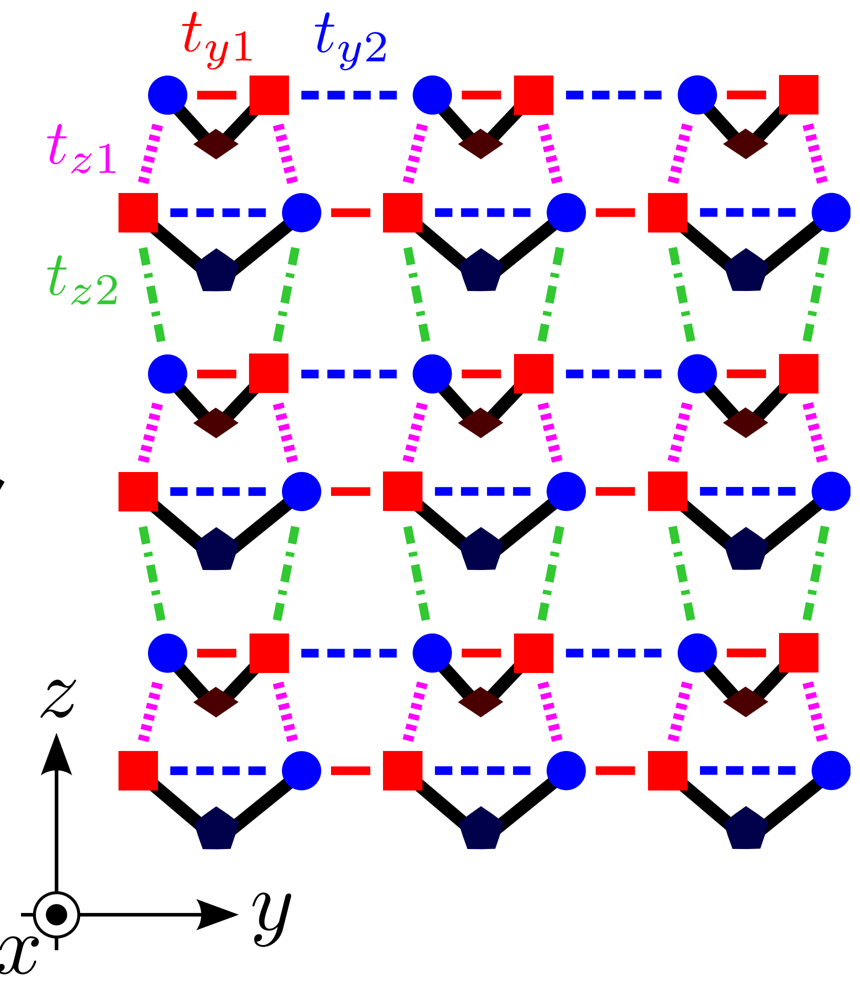


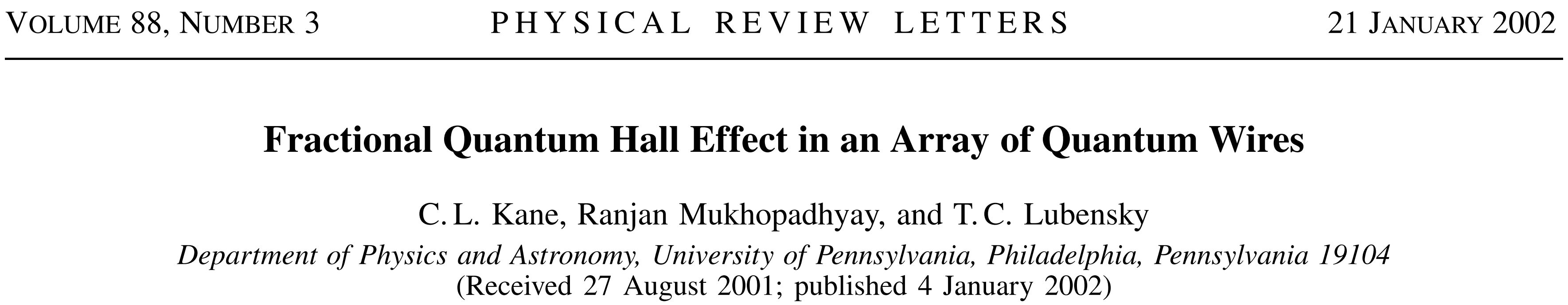
Weyl phase as extended critical phase



Competing couplings of comparable strength

- no gap
- topological 3D phase: Weyl semimetal (WS)





VOLUME 88, NUMBER 3 PHYSICAL REVIEW LETTERS 21 JANUARY 2002

Fractional Quantum Hall Effect in an Array of Quantum Wires

C. L. Kane, Ranjan Mukhopadhyay, and T. C. Lubensky
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 27 August 2001; published 4 January 2002)

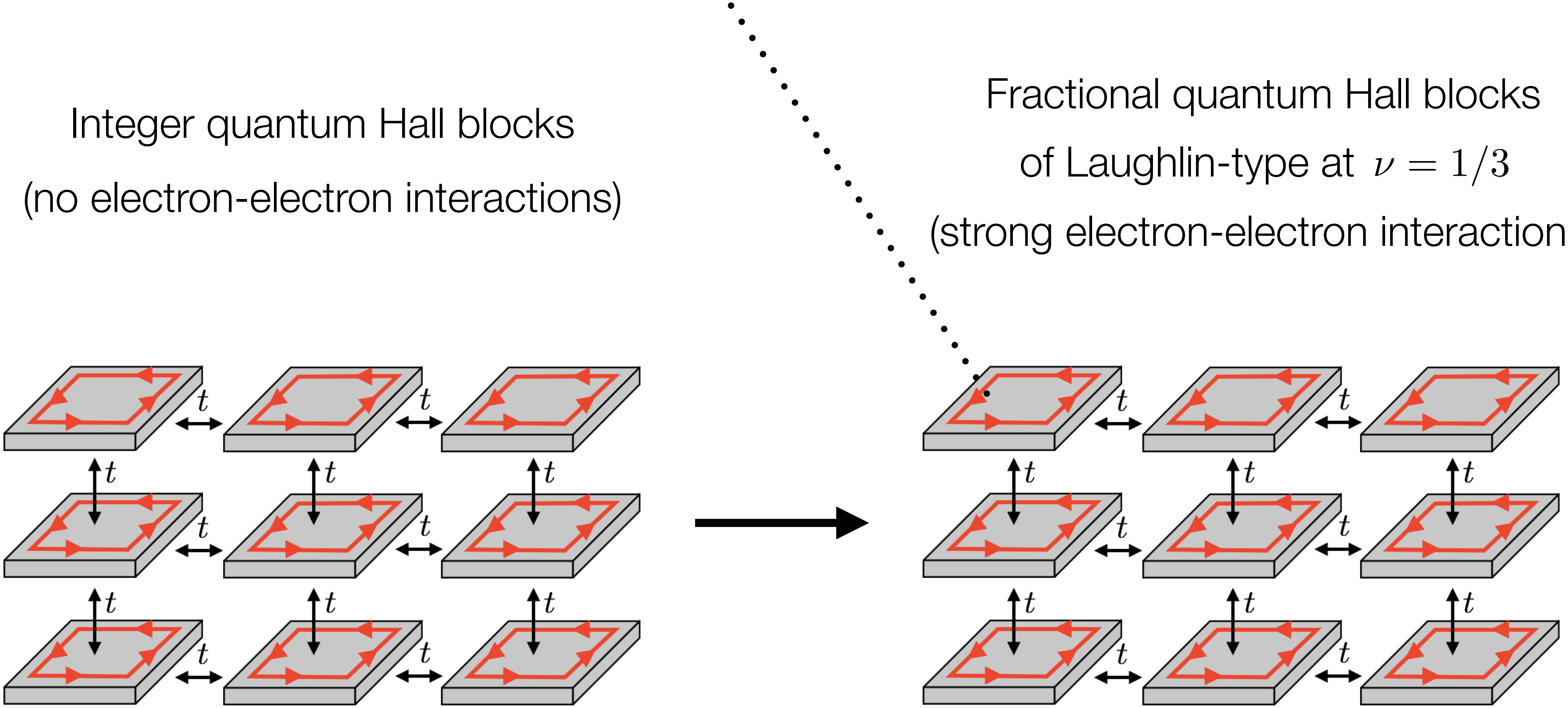
Integer quantum Hall blocks

(no electron-electron interactions)

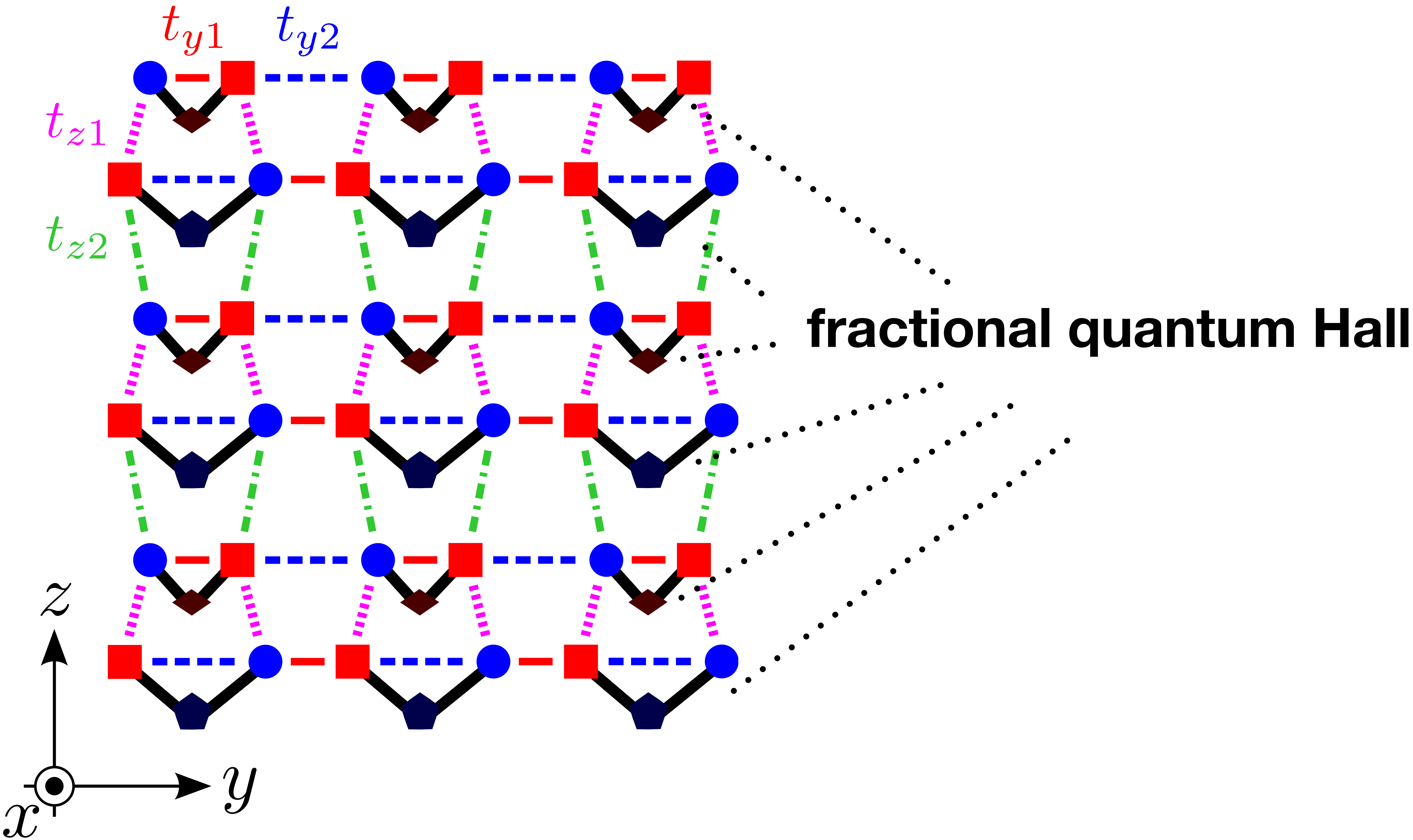
Fractional quantum Hall blocks

of Laughlin-type at $\nu = 1/3$

(strong electron-electron interactions)

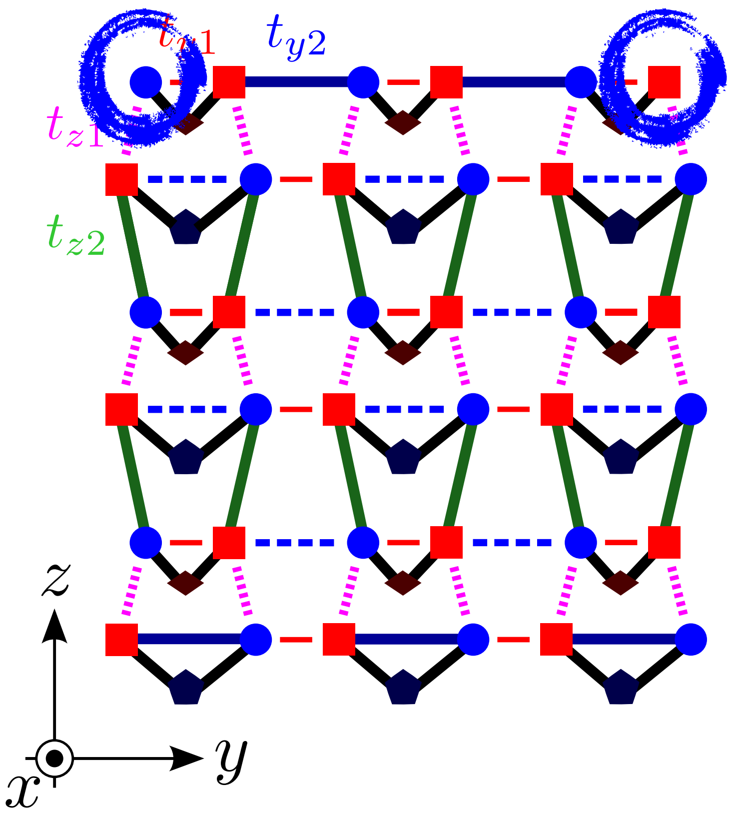


3D system as array of fractional building blocks

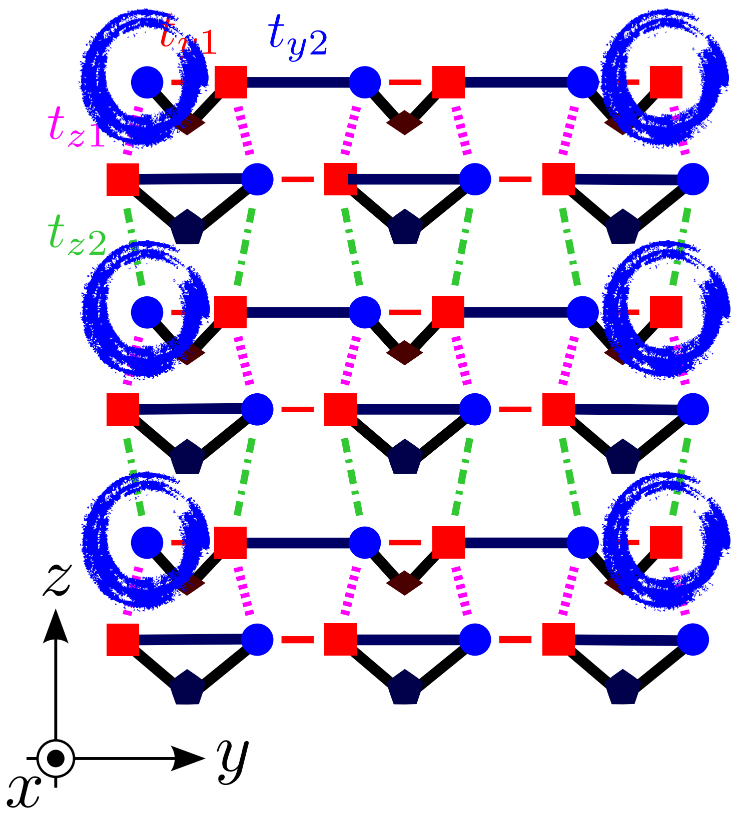


fractional quantum Hall

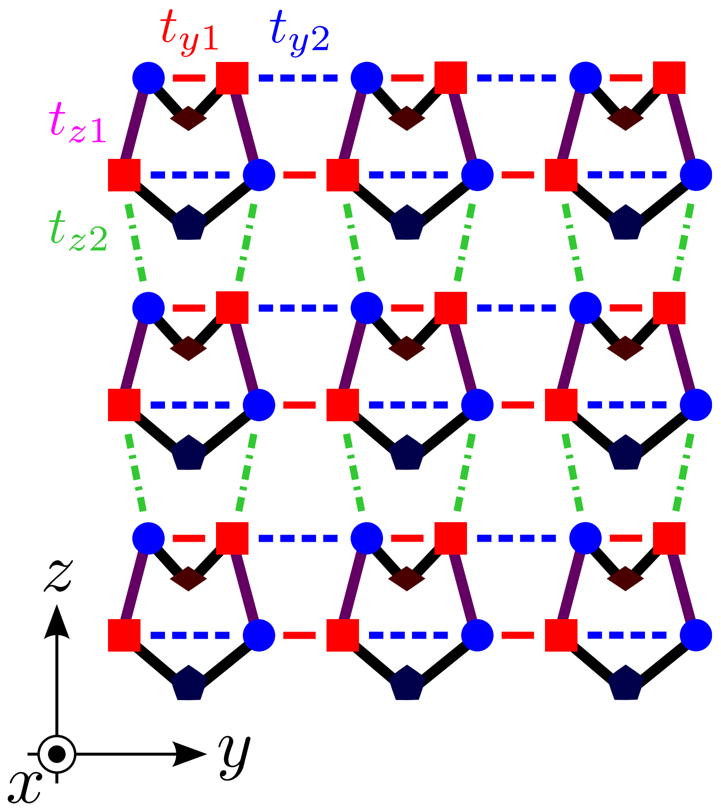
Fractional quantum Hall blocks: limiting cases



Single Surface Fractional QAH
(SSFQAH)



Fractional Quantum Anomalous Hall
(FQAH)

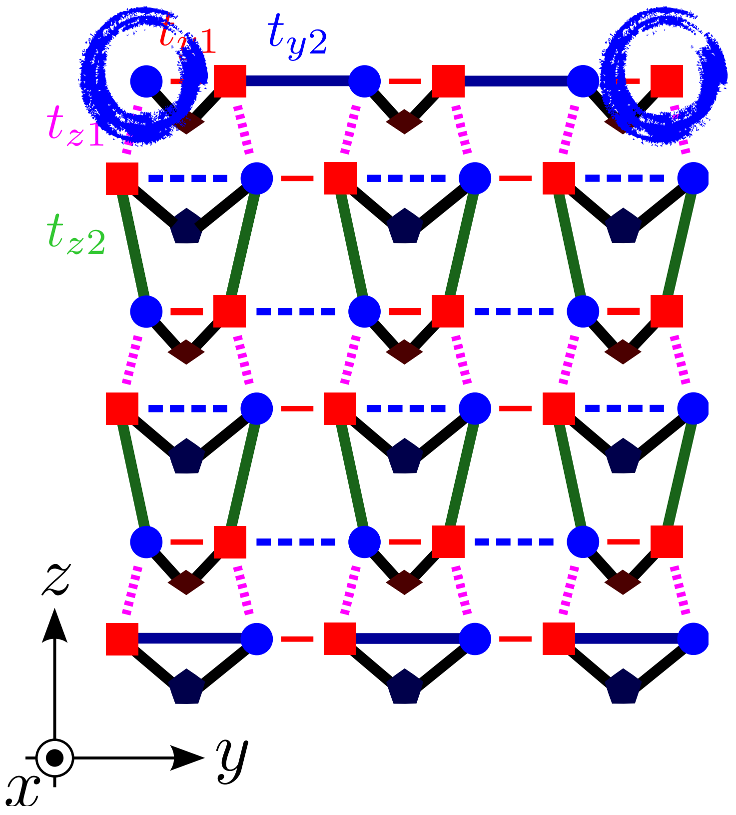


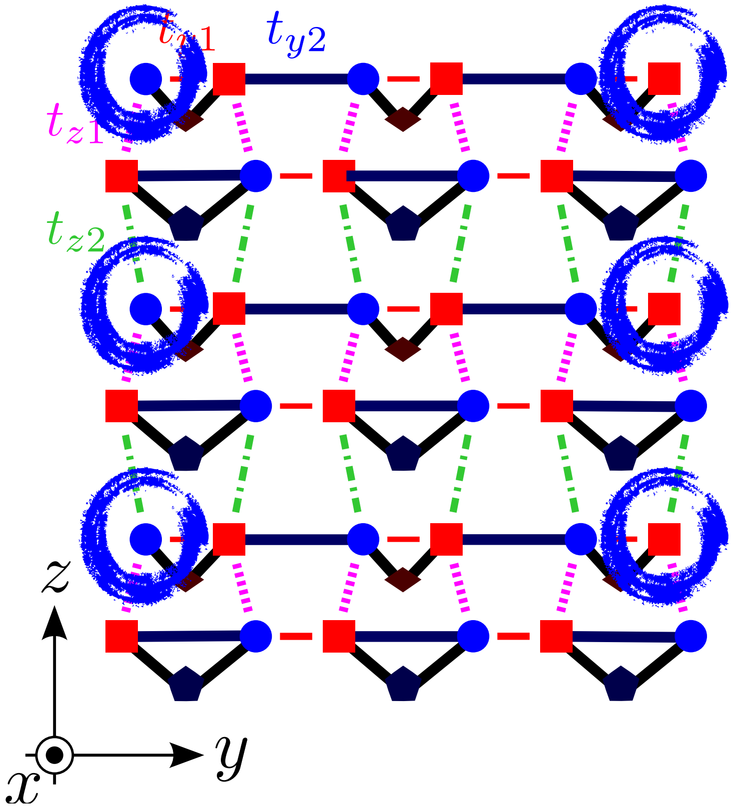
Normal Insulator
(NI)

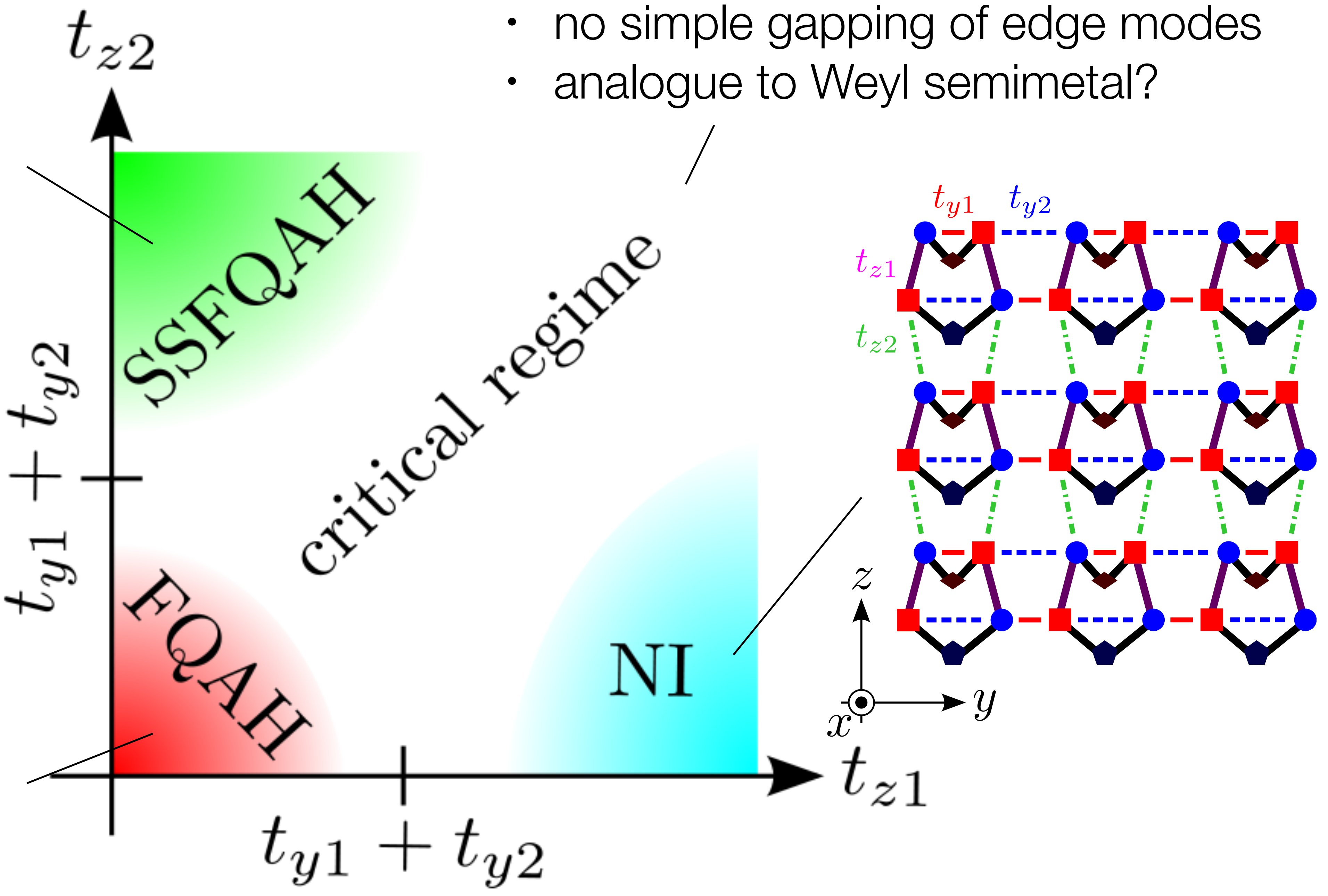
Fractional quantum Hall blocks: limiting cases

Competing couplings of equal strength:

- no simple gapping of edge modes
- analogue to Weyl semimetal?







Factional quantum Hall blocks: limiting cases (2)

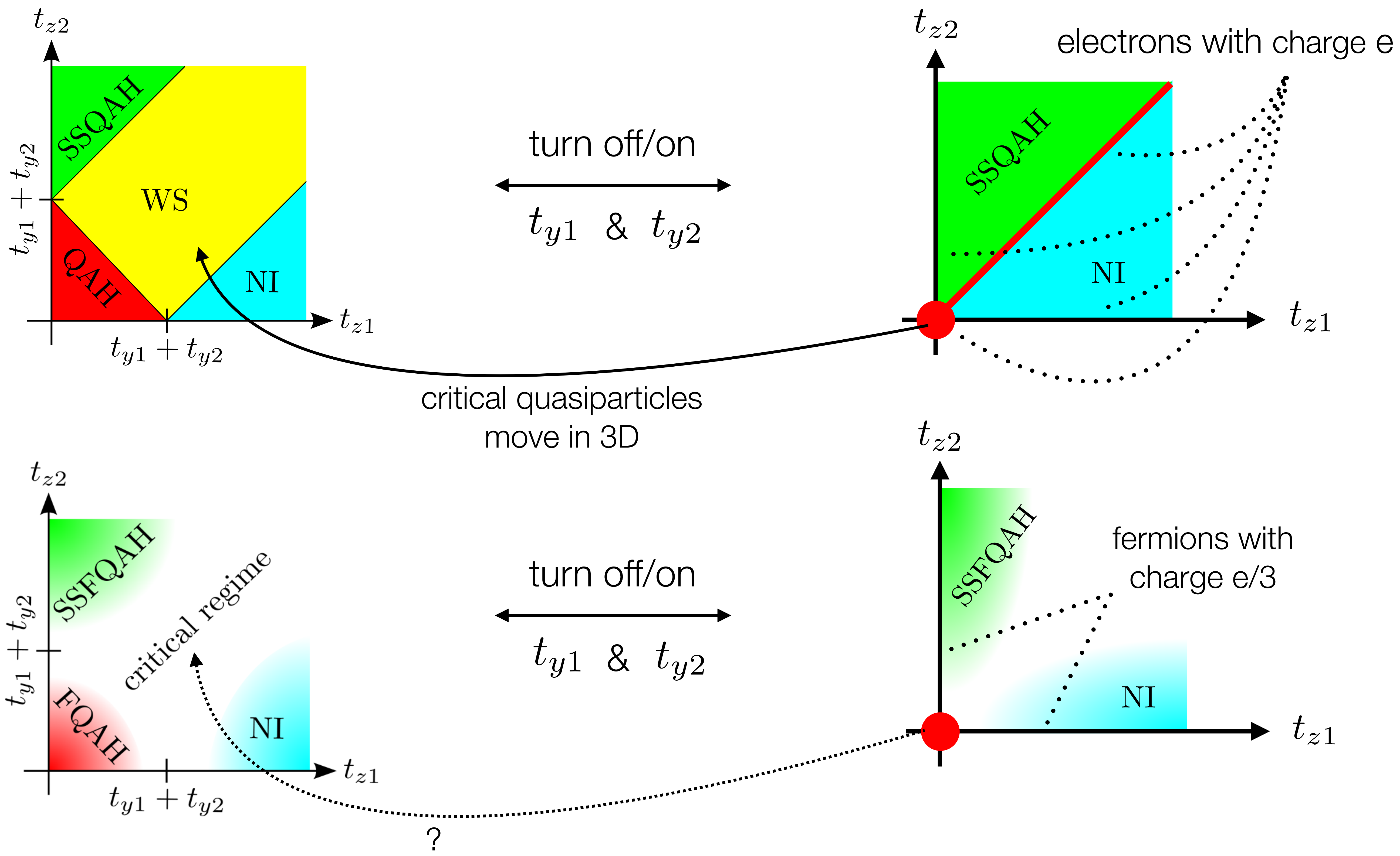
- Consider $t_{z_2} = t_{y1} = t_{y2} = 0$: only cosines due to t_{z1}, no competition
- Then: $$H = H_{\text{quadratic LL}} + H_{\cos}^{(1/3)},$$ $$H_{\cos}^{(1/3)} = \sum_{\text{unit cells}} \int dx \, \frac{t_{z1}}{\pi\alpha} \left[\cos\left(3\phi_A + 3\phi_C + \theta_A - \theta_C\right) + \cos\left(3\phi_B + 3\phi_D + \theta_B - \theta_D\right)\right]$$
- Rewrite as single fermion scattering via refermionization $$H_{\cos}^{(1/3)} = \sum_{\text{unit cells}} \int dx \, t_{z1} \left(\tilde{\Psi}_A^{\dagger} \, \tilde{\Psi}_C + \text{H.c.}\right) + \sum_{\text{unit cells}} \int dx \, t_{z1} \left(\tilde{\Psi}_B^{\dagger} \, \tilde{\Psi}_D + \text{H.c.}\right)$$

with $\Psi_A \sim e^{i(\phi_A - \theta_A)} \longrightarrow \tilde{\Psi}_A \sim e^{i(\tilde{\phi}_A - \tilde{\theta}_A)}$

$$\tilde{\phi}_A = 2\phi_A + \phi_C \qquad \tilde{\theta}_A = \frac{2}{3}\theta_A - \frac{1}{3}\theta_C \qquad [\phi_A(x), \theta_A(x')] = [\tilde{\phi}_A(x), \tilde{\theta}_A(x')]$$

Commutator with $\rho = -\frac{1}{\pi}\sum_i \partial_x \phi_i$ $\Rightarrow$ $\tilde{\Psi}_A$ is canonical fermion with charge e/3

Fractional quantum Hall blocks: critical phase?



electrons with charge e

turn off/on

t_{y1} & t_{y2}

critical quasiparticles
move in 3D

turn off/on

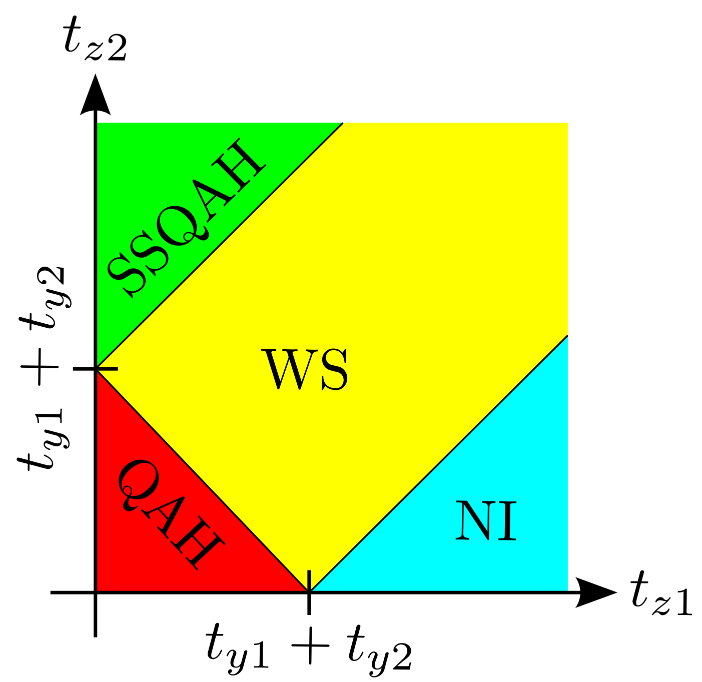
t_{y1} & t_{y2}

fermions with
charge e/3

?

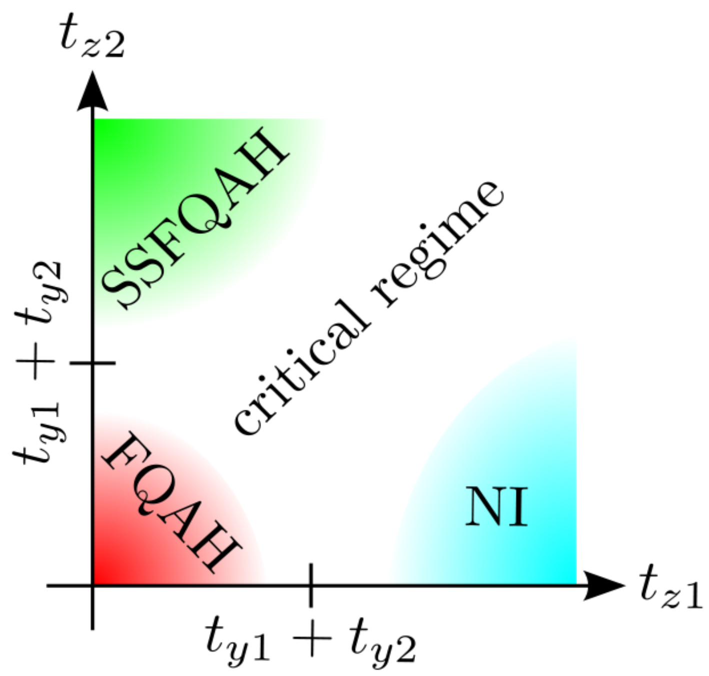
Fractional quantum Hall blocks: critical phase?

- Study $H = H_{\text{quadratic LL}} + H_{\cos}^{(\nu)}$ using $\cos(a-b) = \frac{1}{2}\left(e^{ia}\,e^{-ib} + \text{H.c.}\right)$



$$\Rightarrow \quad H_{\cos}^{(\nu)} = \int dx \sum_{\text{unit cells } i,j} \Psi_i^{(\nu)\dagger}\,\mathcal{H}_{ij}\,\Psi_j^{(\nu)}$$

sine-Gordon prefactors $t_{y,z,1,2}$:
can be adjusted as identical



$$\Psi_i^{(\nu=1)} \sim \begin{pmatrix} e^{-i(\phi_{Ai}-\theta_{Ai})} \\ e^{-i(-\phi_{B}-\theta_{Bi})} \\ e^{-i(-\phi_{Ci}-\theta_{Ci})} \\ e^{-i(\phi_{Di}-\theta_{Di})} \end{pmatrix} \qquad \Psi_i^{(\nu=1/3)} \sim \begin{pmatrix} e^{-i(3\phi_{Ai}-\theta_{Ai})} \\ e^{-i(-3\phi_{B}-\theta_{Bi})} \\ e^{-i(-3\phi_{Ci}-\theta_{Ci})} \\ e^{-i(3\phi_{Di}-\theta_{Di})} \end{pmatrix}$$

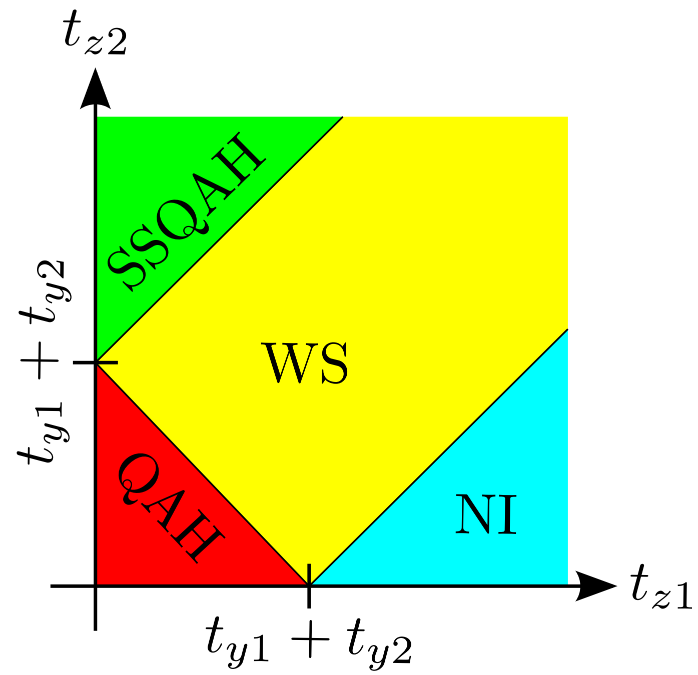
(canonical fermions)

$(\Psi_{R,L} \sim e^{i(\pm\phi-\theta)})$

(not canonical fermions)

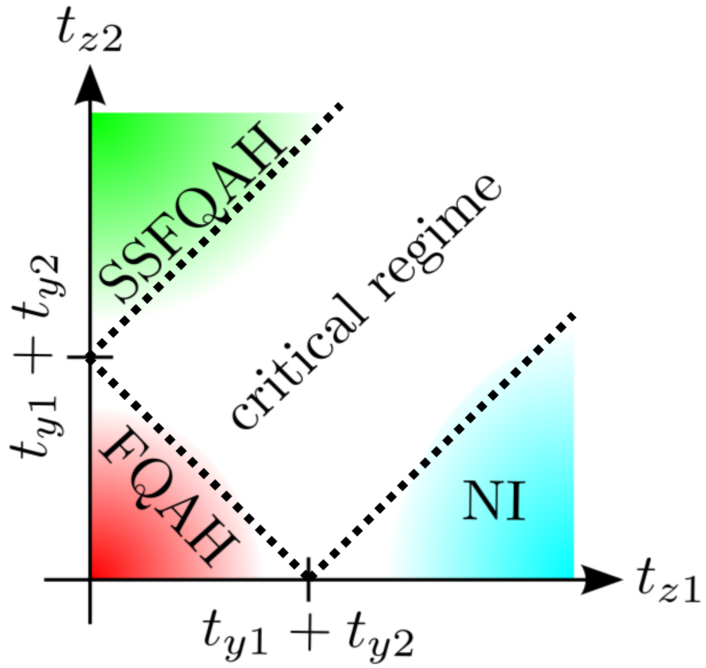
Fractional quantum Hall blocks: critical phase?

- Study $H = H_{\text{quadratic LL}} + H^{(\nu)}_{\cos}$ using $\cos(a-b) = \frac{1}{2}\left(e^{ia}\, e^{-ib} + \text{H.c.}\right)$



$$\Rightarrow \quad H^{(\nu)}_{\cos} = \int dx \sum_{\text{unit cells } i,j} \Psi^{(\nu)\dagger}_i\, \mathcal{H}_{ij}\, \Psi^{(\nu)}_j$$

sine-Gordon prefactors $t_{y,z,1,2}$:
can be adjusted as identical



Fourier transformation: $H^{(\nu)}_{\cos} = \sum_{\vec{k}} \Psi^{(\nu)\dagger}_{\vec{k}}\, \mathcal{H}_{\vec{k}}\, \Psi^{(\nu)}_{\vec{k}}$

$\exists\ \Psi^{(\nu)}_{\vec{k}=\text{Weyl node}}$ which has $\mathcal{H}_{\vec{k}=\text{Weyl node}} = 0$

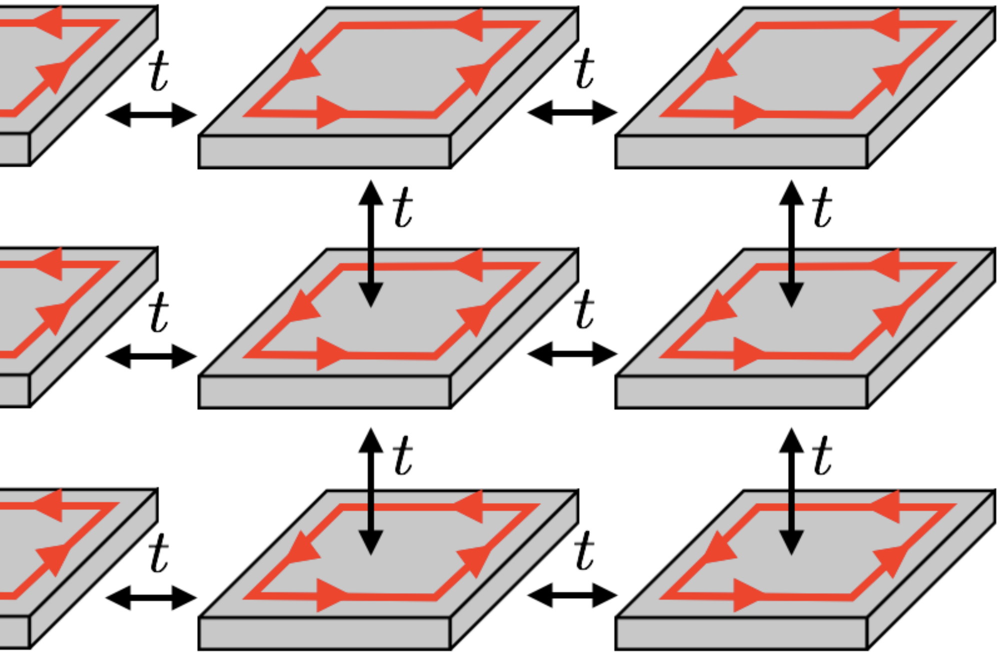
! $\Psi^{(\nu=1/3)}_{\vec{k}}$ not a fermion: what is $\Psi^{(\nu=1/3)}_{\vec{k}=\text{Weyl node}}$?

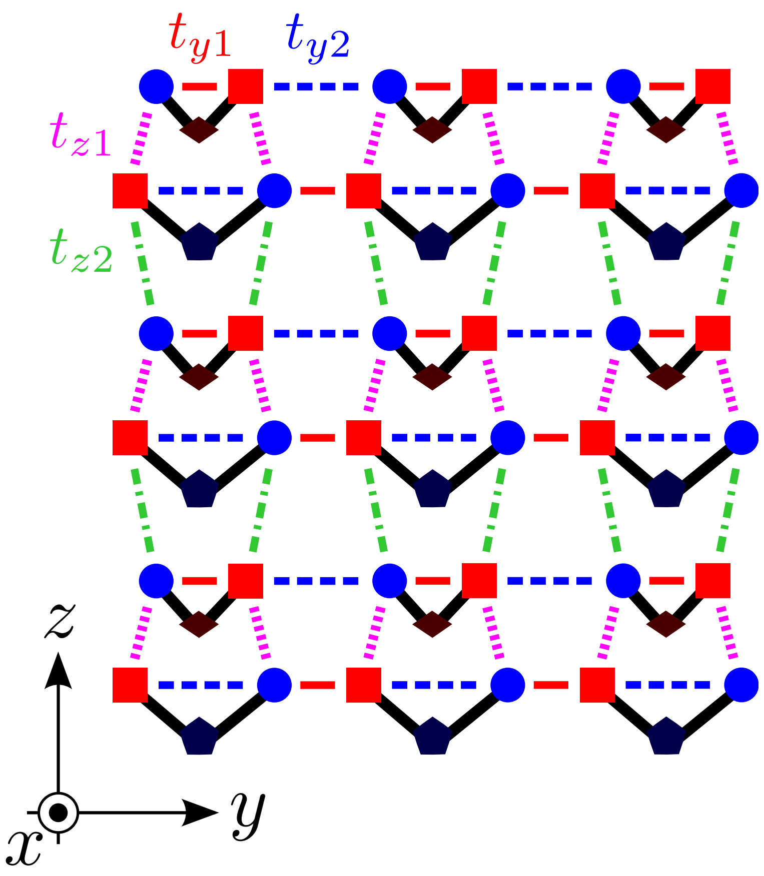
Summary

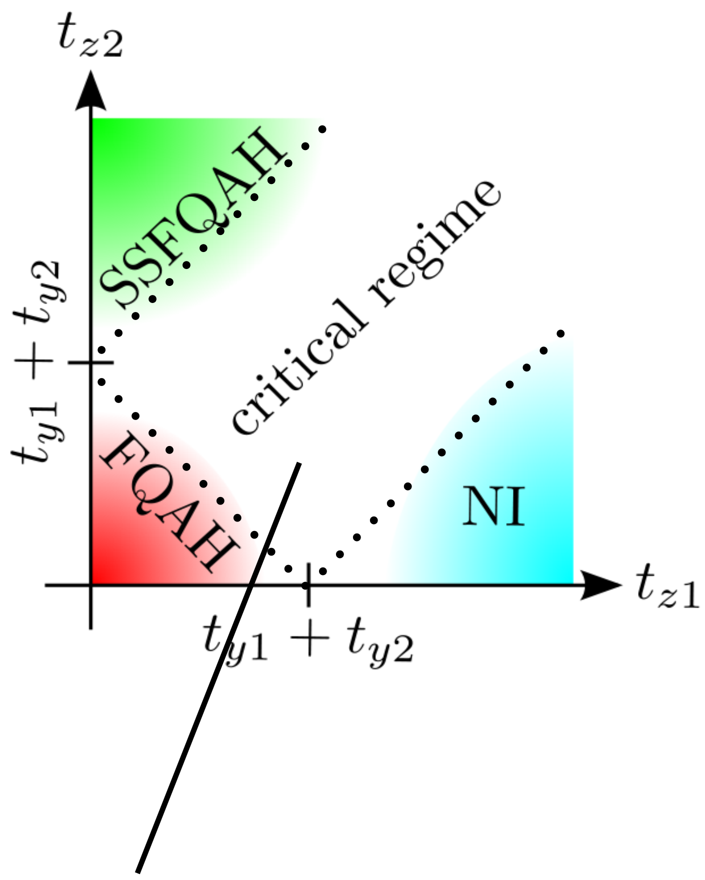
Quantum LEGO:
couple edge modes of 2D of building blocks
to obtain 3D system

⇒

new tool for engineering
and analysis of interacting
topological 3D systems







Gapless phase of fractionally charged fermions? In progress!