Carbon Nanotube Quantum Dot: Realization of Two-Channel Kondo Effect

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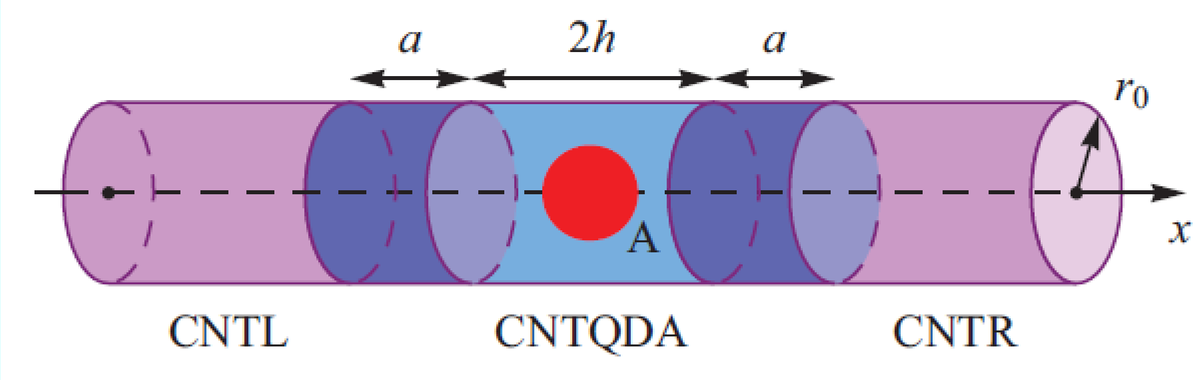
Tunneling Junction

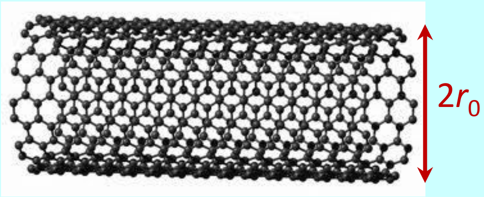


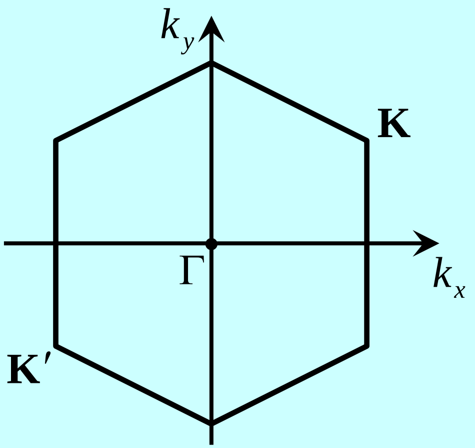
- CNTL and CNTR are left and right metallic leads
- CNTQDA is a carbon nanotube quantum dot of length 2*h* with caged atom
- Two tunnel barriers of length *a*

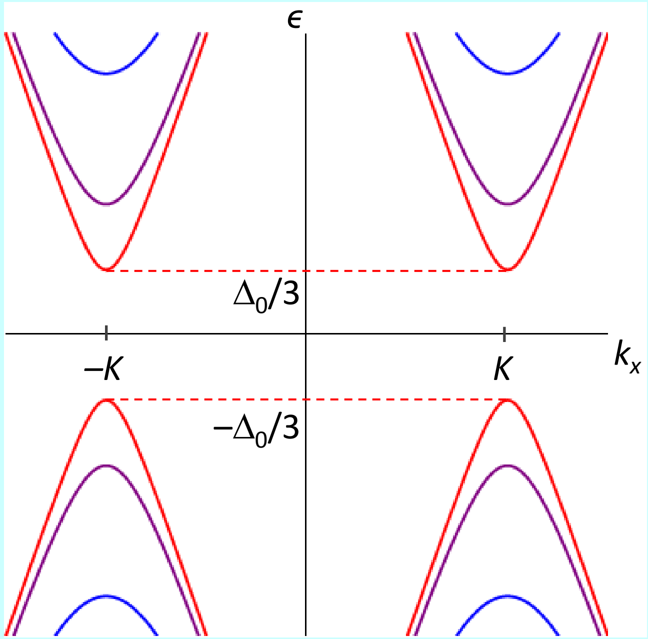
Outline

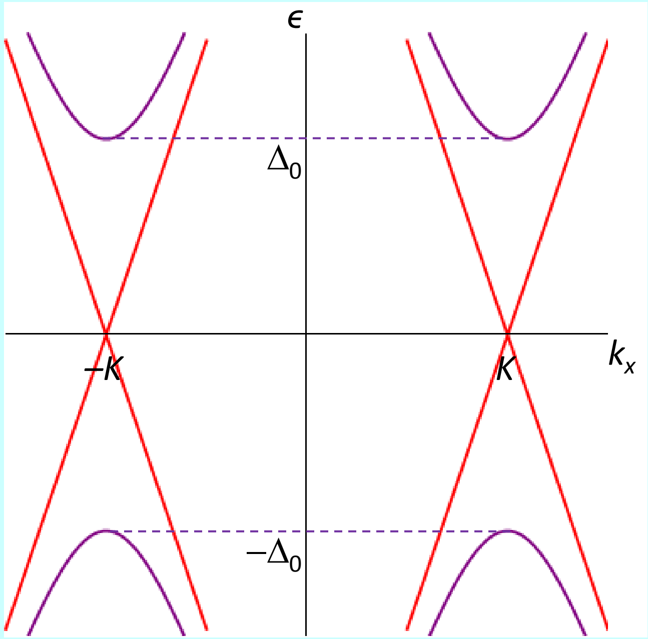
- Electronic properties of Carbon Nanotubes
- Construction of tunnel junction consisting of two metallic leads and a QD with an atom on the CNT axis
- Kondo Hamiltonian
- Two regimes of the poor man’s scaling procedure
- Conductance, entropy and specific heat
- Conclusions

Carbon Nanotubes





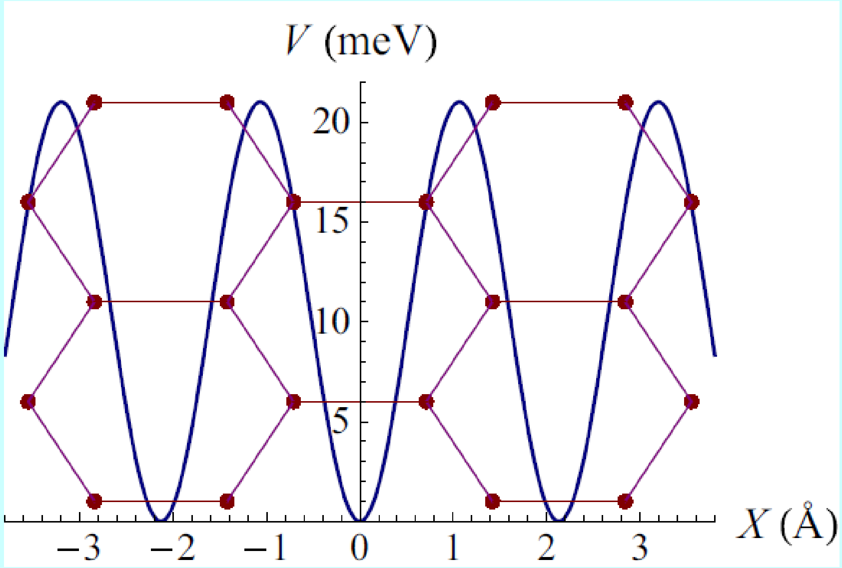


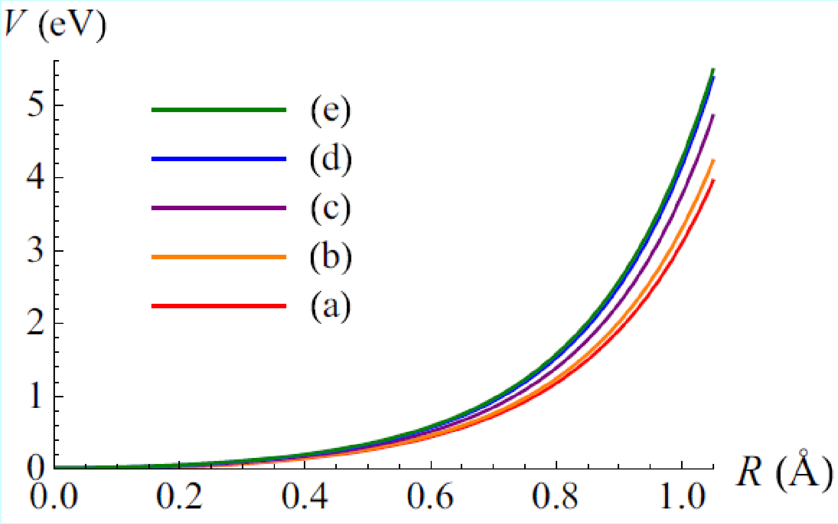


- Two valleys near the points K and K′ of the first Brillouin zone
- Degeneracy in both spin (↑, ↓) and isospin (or valley **K**, **K′**) quantum numbers
- The energy spectrum of a semiconductor CNT is

$$\varepsilon_{mq} = \pm\sqrt{(\hbar v q)^2 + \Delta_0^2\left(m + \frac{1}{3}\right)^2}, \quad \Delta_0 = \frac{\hbar v}{r_0}$$

Implanting an Atom on the CNT Axis





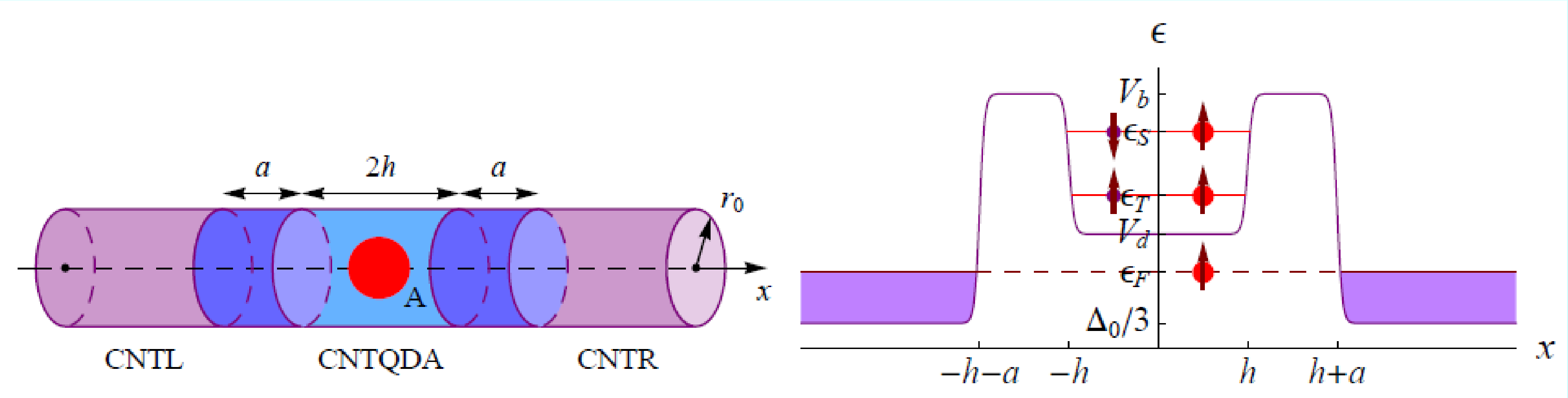
- Van der Waals interaction between the caged atom and the CNT possesses minima and saddle points at

$$X_n^{\min} = 2nX_1,$$
$$X_n^{\text{sad}} = (2n+1)X_1,$$
$$X_1 = \frac{3a_0}{4\sqrt{3}}.$$

- Neighboring minima are separated by tunnel barriers of height W_b = 21.02 meV.

- The caged atom performs small oscillations around the point **R** = (0,0).

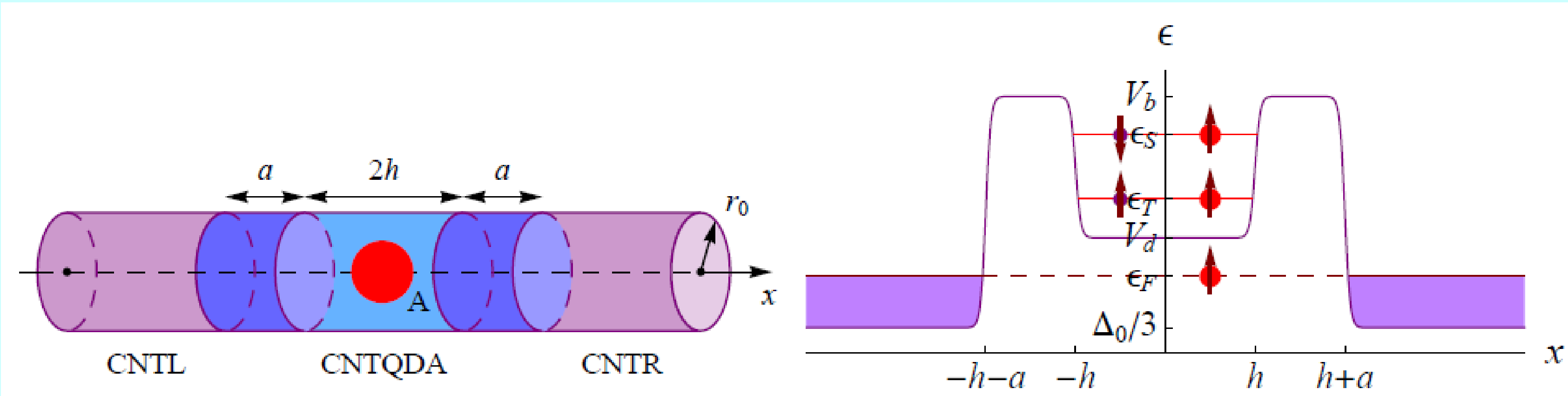
Tunnel Junction



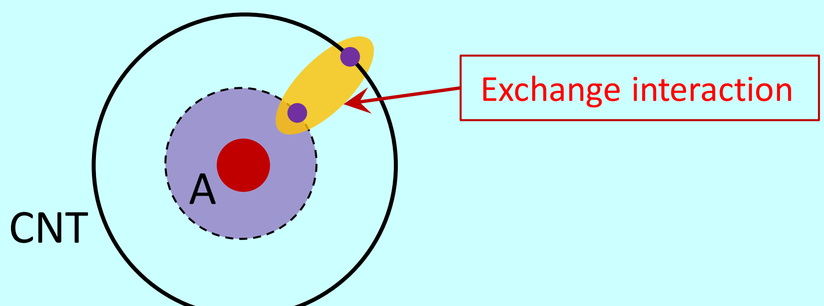
$$\Delta_g(x) = -\frac{\Delta_0}{3} - V_0\,\Theta(|x| - h - a) + V_d\,\Theta(h - |x|) + V_b\,\Theta(|x| - h)\Theta(h + a - |x|),$$

- Gate voltage is such that $\Delta_0/3 < \epsilon_F < V_d < V_b$.
- The CNT is divided on left and right leads, and the quantum dot separated one from another by the barriers.

Energy Levels of CNT(QD)



- Fermi energy is below the single-electron level.
- The ground state of the QD hosts solely the spin-1/2 caged atom.
- The excited states contain one electron and the caged atom. Together, they form singlet (S) and triplet (T) states.
- Exchange interaction between the atom and electron gives the singlet-triplet splitting with $\epsilon_S > \epsilon_T$



Kondo Hamiltonian

$$H_K = \sum_{\xi q q'} \left(K_{\varepsilon_q \varepsilon_{q'}} n_{\xi, qq'} + J_{\varepsilon_q \varepsilon_{q'}} \left(\mathbf{S} \cdot \mathbf{s}_{\xi, qq'} \right) \right),$$

- $n_{\xi,qq'}$ is the electron density operator
- $\mathbf{s}_{\xi,qq'}$ is the spin density operator
- **S** is the spin of the quantum dot

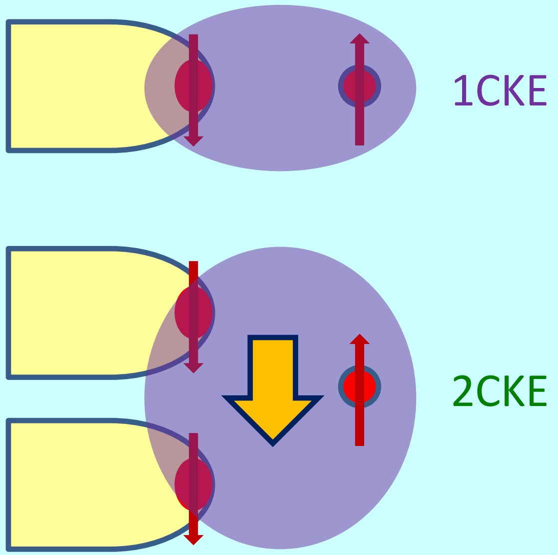
$$K_{\varepsilon\varepsilon'} = \frac{3 t_\varepsilon t_{\varepsilon'}}{4(\varepsilon_T - \varepsilon_F)} + \frac{t_\varepsilon t_{\varepsilon'}}{4(\varepsilon_S - \varepsilon_F)}, \quad J_{\varepsilon\varepsilon'} = \frac{t_\varepsilon t_{\varepsilon'}}{\varepsilon_T - \varepsilon_F} - \frac{t_\varepsilon t_{\varepsilon'}}{\varepsilon_S - \varepsilon_F},$$

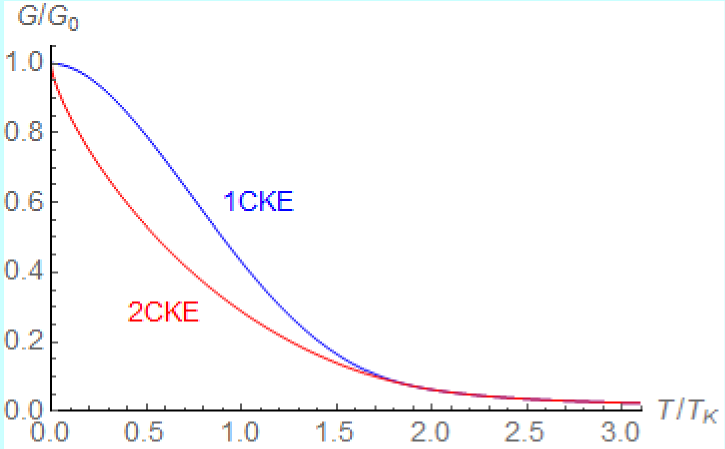
t_ε is a tunneling rate

- $\varepsilon_S > \varepsilon_T$ ⇨ $J_{\varepsilon_q \varepsilon_{q'}} > 0$ ⇨ the exchange interaction is antiferromagnetic

- ξ is a good quantum number ⇨ **two channel Kondo effect**

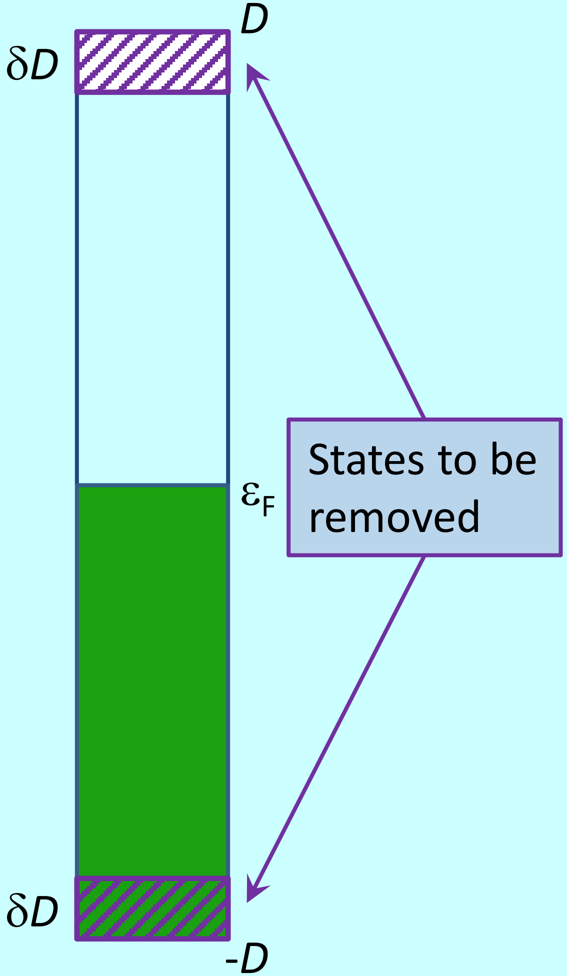
Single-Channel and Two-Channel Kondo Effect



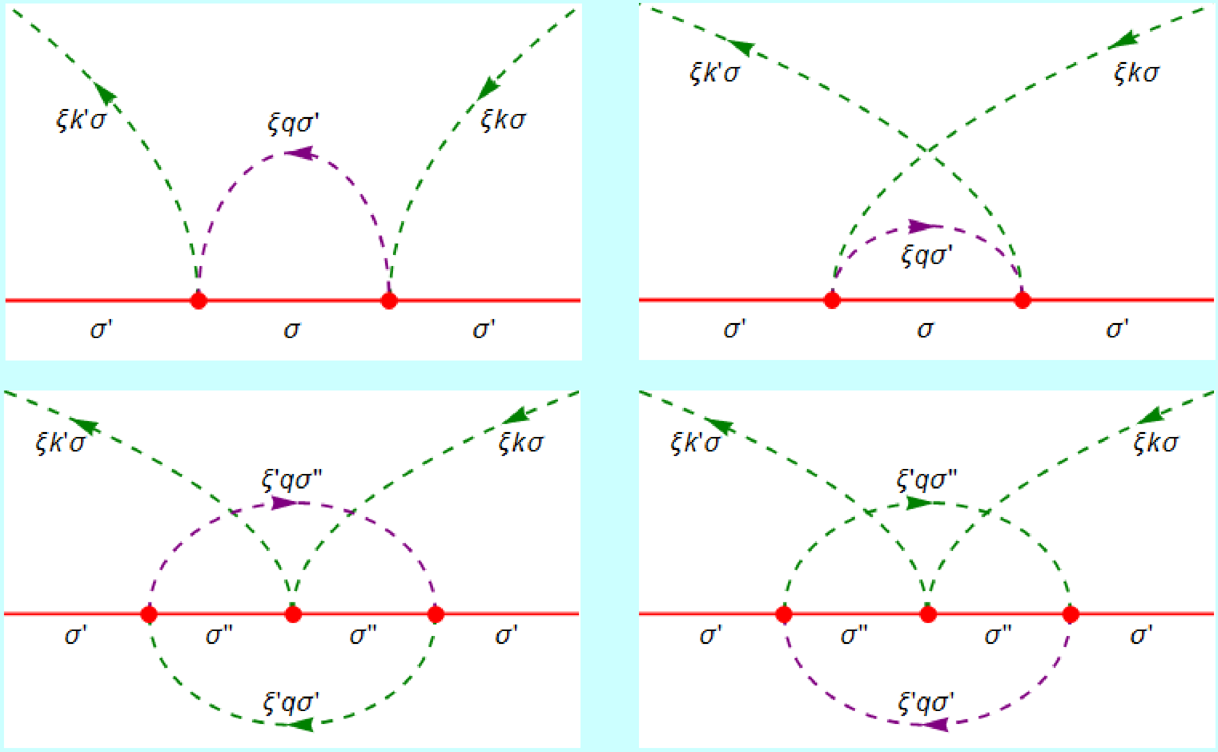


- Single channel Kondo effect: the spin of the dot is fully screened by the conduction band electrons
- Conductance behaves as T^2 – Fermi liquid fixed point
- Two channel Kondo effect: the spin of the dot is over-screened by the conduction band electrons
- Conductance behaves as $T^{1/2}$ – non-Fermi liquid fixed point
- Effect of the two channel Kondo effect is observable just in the strong coupling limit $T < T_K$

Poor Man's Scaling Technique

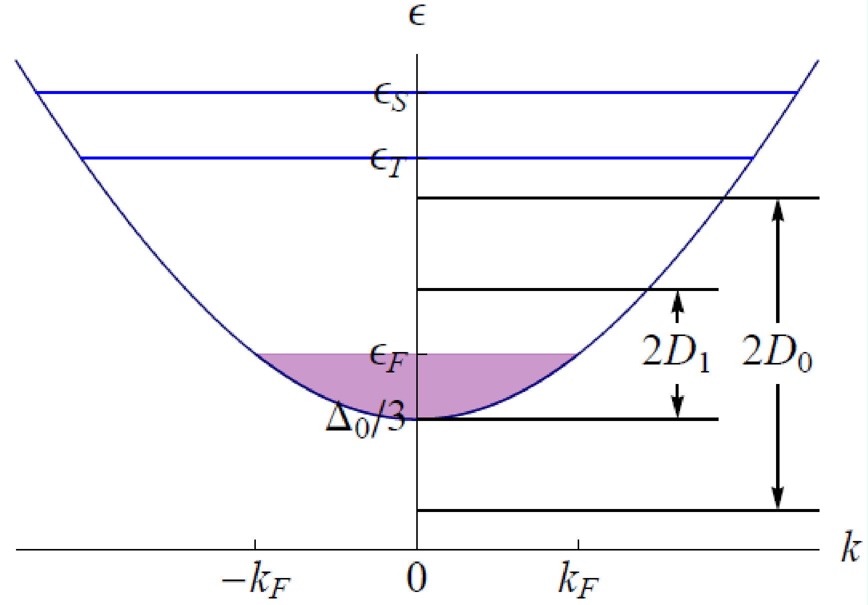


- Strong correlation between the QD and the leads – the perturbation theory cannot be applied



$$\boxed{D \rightarrow D' = D - 2\delta D \quad \Rightarrow \quad K \rightarrow K',\ J \rightarrow J'}$$

Two Regimes of the Poor Man's Scaling Procedure

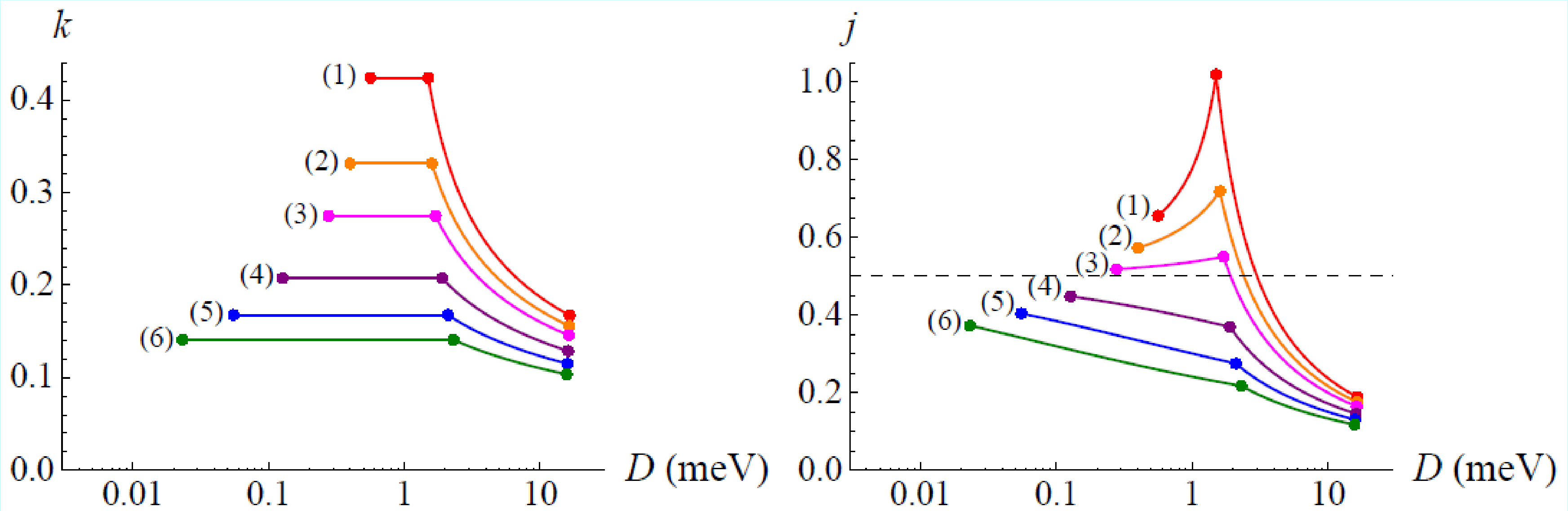


$D_0 > \epsilon_F - \Delta_0/3$: There are two different regimes of the poor man's scaling procedure

- <u>$D_0 > D > D_1 = \epsilon_F - \Delta_0/3$</u>:
$\epsilon_F - D$ is below the bottom of the conduction band

- <u>$D < D_1$</u>:
$\epsilon_F - D$ is above the bottom of the conduction band

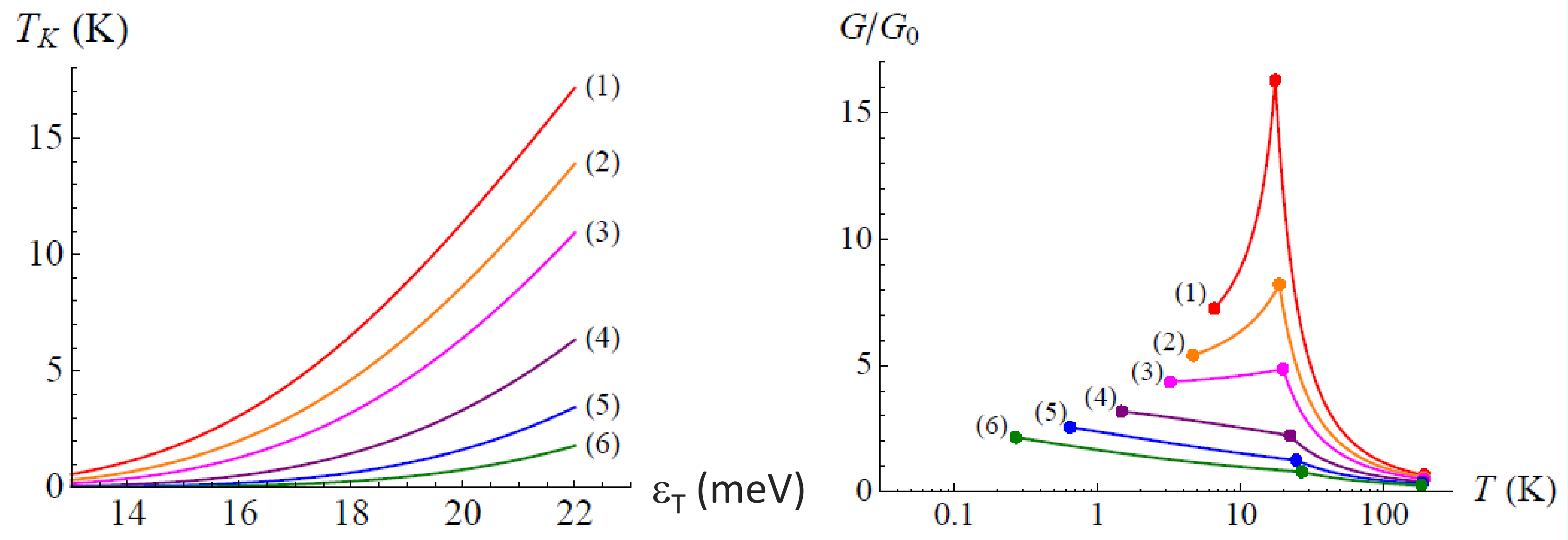
Scaling of Couplings



k and *j* as functions of *D* for ε_T = 18 meV and different values of ϵ_F. Here $\varepsilon_S - \varepsilon_T$=120meV, curves (1)–(6) correspond to ϵ_F = 1.5, 1.6, 1.7, 1.9, 2.1, and 2.3 meV, respectively.

Note the nonmonotonic behavior of the curves (1), (2) and (3) [$\epsilon_F \leq 1.7$ meV]:
When $D > D_1$, the effective coupling $j\,(D)$ increases to the value over j^*, and then within the interval $D < D_1$, $j(D)$ decreases approaching j^*.

Kondo Temperature and Conductance



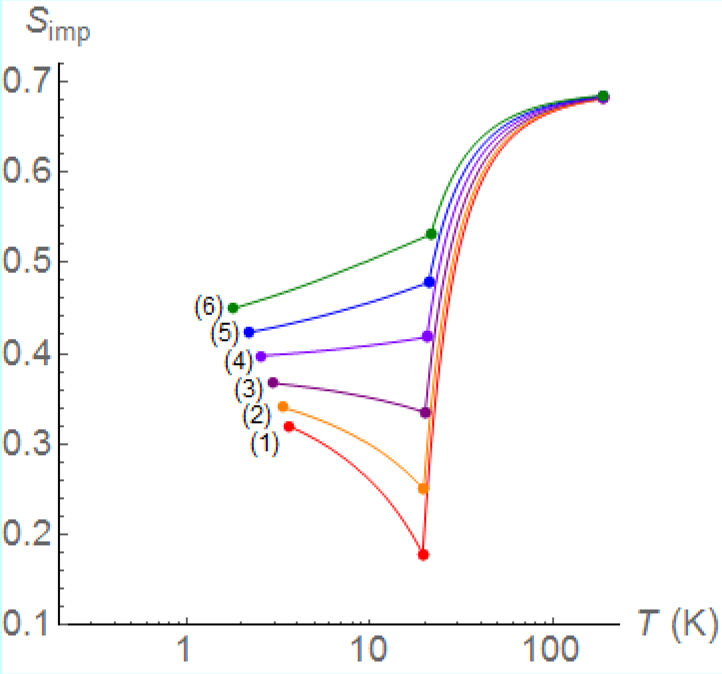
- T_K as a function of ε_T and different values of ϵ_F .
- The conductance $G(T)$ as function of T for ϵ_T = 18 meV and different values of ϵ_F.

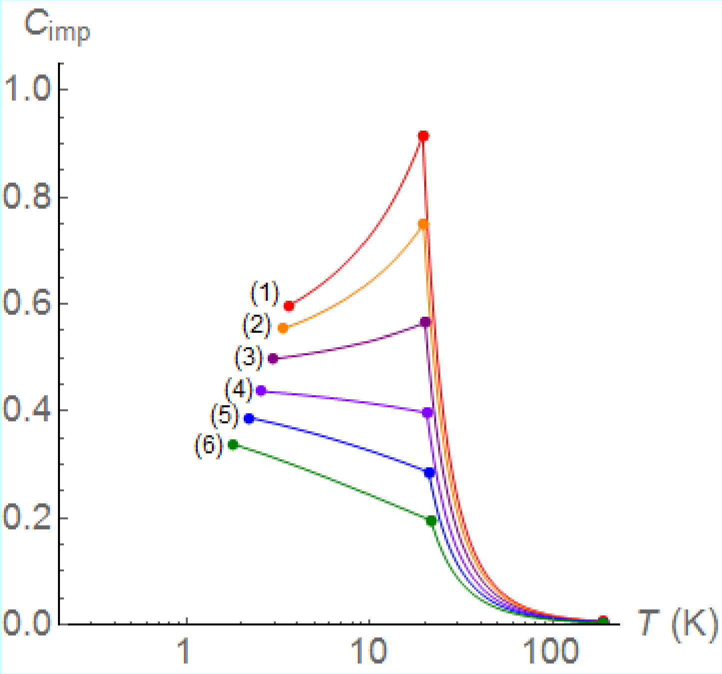
Here curves (1)–(6) correspond to ϵ_F = 1.5, 1.6, 1.7, 1.9, 2.1, and 2.3 meV, respectively.

$$G = \frac{\pi^2 G_0}{2}\left\{k^2(T) + 3j^2(T)\right\}, \quad G_0 = \frac{e^2}{\pi\hbar}.$$

Note the nonmonotonic behavior of G(T) for $\epsilon_F \leq 1.7$ meV [curves (1)–(3)]. This exotic behavior is caused by the nonmonotonicity of $j\,(T\,)$.

Entropy and specific heat





- The entropy $S_{\text{imp}}(T)$ as function of T for ϵ_T = 18 meV and different values of ϵ_F.

Here curves (1)–(6) correspond to ϵ_F = 1.5, 1.6, 1.7, 1.9, 2.1, and 2.3 meV, respectively.

$$S_{\text{imp}} = \ln 2 - \frac{\pi^2}{4} j^3(T), \qquad C_{\text{imp}} = \frac{3\pi^2}{4} j^4(T).$$

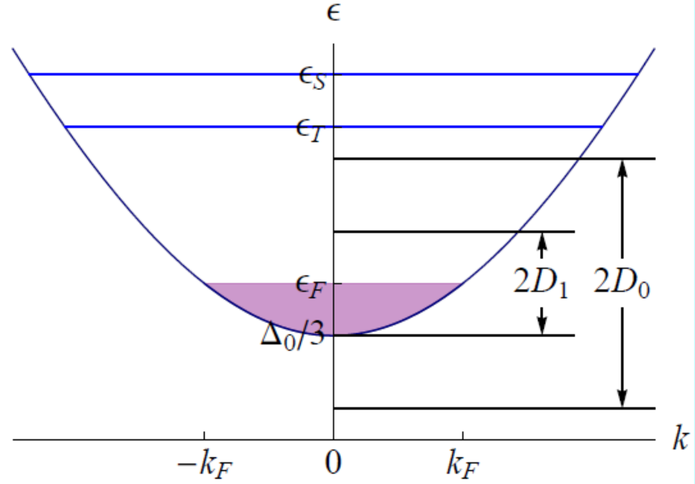
Conclusions

- We demonstrate the possibility of tuning a semiconductor CNT into a tunnel junction consisting of two CNT metallic leads and a CNT quantum dot
- Isospin (the valley quantum number) is a good quantum number that results in two channel Kondo effect
- The proposed device reveals two scaling regimes in which the running coupling constant behaves differently
- Nonmonotonic behavior of the conductance, entropy and specific heat as function of temperature enables the physics of the 2CKE to be visible also in the weak coupling regime

Scaling Equations:
First Interval

The scaling equations are,

$$\frac{\partial k}{\partial \ln D} = -k^2 - \frac{3j^2}{16}, \quad \frac{\partial j}{\partial \ln D} = -2kj - \frac{j^2}{2},$$



$D_0 > D > D_1$, $k(D_0) = k_0$, $j(D_0) = j_0$.

Solution of the scaling equations is,

$$k(D) = \frac{1}{4\ln\left(\frac{D}{T_g}\right)} + \frac{3}{4\ln\left(\frac{D}{T_f}\right)}, \quad j(D) = \frac{1}{\ln\left(\frac{D}{T_g}\right)} + \frac{1}{\ln\left(\frac{D}{T_f}\right)},$$

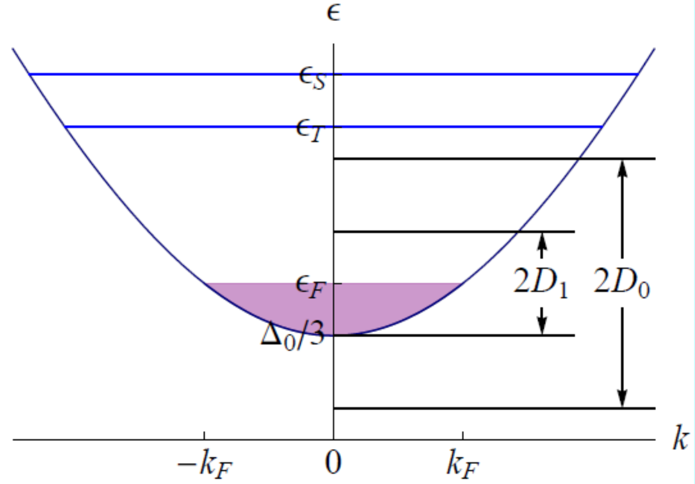
where

$$T_g = D_0 \exp\left(-\frac{4}{4k_0 + 3j_0}\right), \quad T_f = D_0 \exp\left(-\frac{4}{4k_0 - j_0}\right).$$

Scaling Equations:
Second Interval

The scaling equation is,

$$\frac{\partial j}{\partial \ln D} = -j^2 + 2j^3, \quad D < D_1, \quad j(D_1) = j_1.$$



Solution of the scaling equation is,

$$\frac{1}{j_1} - \frac{1}{j} + 2\ln\left(\frac{j(1-2j_1)}{j_1(1-2j)}\right) = \ln\left(\frac{D_1}{D}\right).$$

The scaling invariants are

$$T_K = D_1 \exp\left(-\frac{1}{j_1}\right), \quad T^* = D_1 \exp\left(-\frac{1}{j^*}\right), \quad j^* = \frac{1}{2}.$$