Dynamics in dissipative quantum systems

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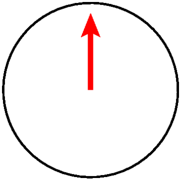
Motivation

"The simplest model involving quantum coherence is a two-state system.[...] In reality, such a system is strongly affected by the surroundings"

– Quantum dissipative systems, U. Weiss

how we use "coherence"

-in a small quantum system



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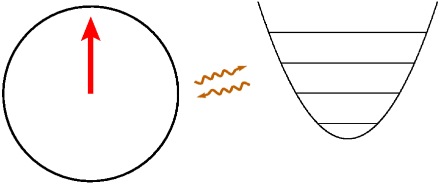
how we use "coherence"

- in a small quantum system coupled to a large environment



- spin-boson model: widely used in physics, chemistry and quantum information sciences
- here: unbiased, ohmic spin boson model in scaling limit

Goal and Approach

- dissipative system: relaxation dynamics in coupling(α)-T-plane

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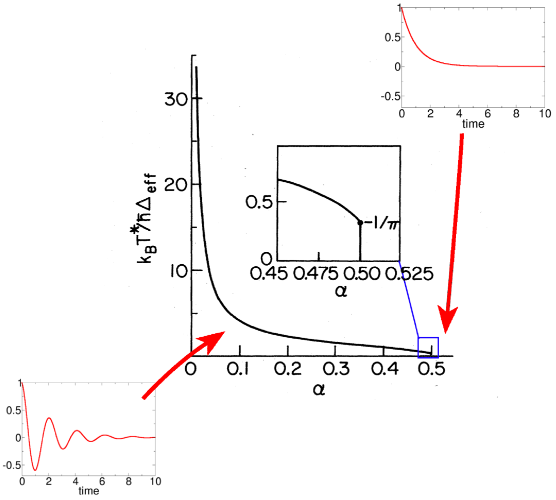
Garg '85; Weiss, Grabert '86; Leggett et al. '87



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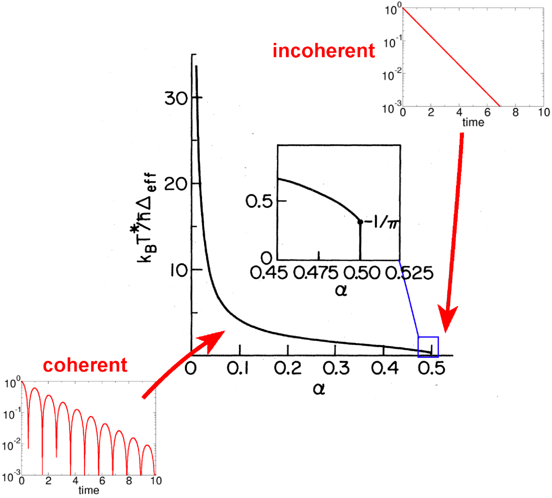

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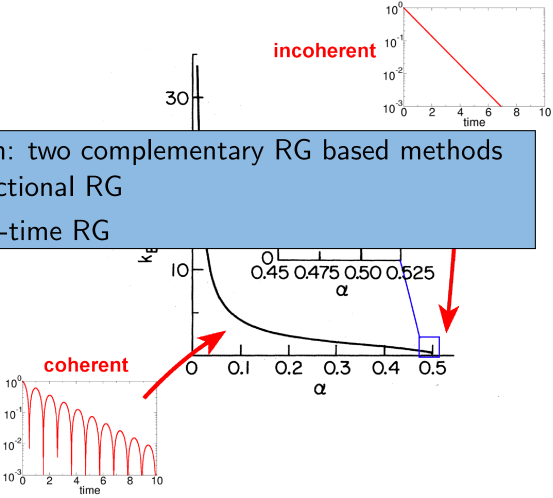


approach: two complementary RG based methods

- functional RG
- real-time RG

Model

Hamiltonian

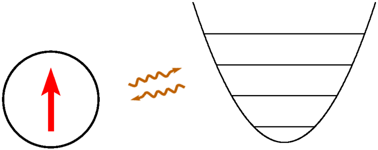
$$H = -\frac{\Delta}{2}\sigma_x - \sum_k \frac{\lambda_k}{2}\sigma_z \left(b_k^\dagger + b_k\right) + \sum_k \omega_k b_k^\dagger b_k$$

coupling: spectral density

$$J(\omega) = \sum_k \lambda_k^2 \delta(\omega - \omega_k)$$

ohmic case: rich physics

$$J(\omega) = 2\alpha\omega\Theta(\omega_c - \omega)$$



initial state: spin-up $\times$ therm. eq.

Leggett et al. '87, Weiss '12

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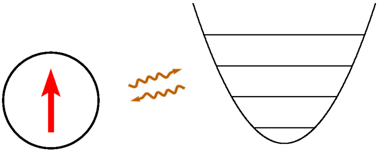

initial state: spin-up $\times$ therm. eq.

scaling limit: $\omega_c \to \infty$

emergent (Kondo) scale: $T_K = \Delta(\Delta/\omega_c)^{\alpha/(1-\alpha)}$

Leggett et al. '87, Weiss '12

Model: $|\alpha - 1/2| \ll 1$

dynamics: coherent (osz. damped) $\leftrightarrow$ incoherent (monotonic)

spin expect. value $P(t) = \langle \sigma_z(t) \rangle$: $\alpha < 1/2$ coh., $1/2 < \alpha < 1$ incoh.

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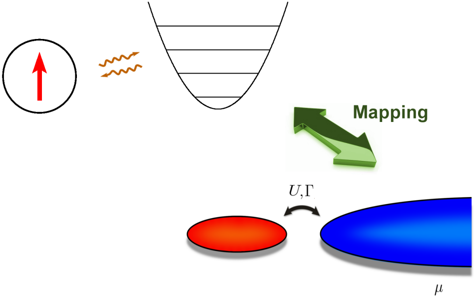
bosonization: maps to interacting resonant level model (IRLM)

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \sqrt{\frac{\Gamma_0}{2\pi\nu}} \sum_k (d^\dagger a_k + d a_k^\dagger) + \frac{U}{2\nu}(d^\dagger d - d d^\dagger) \sum_{kk'} : a_k^\dagger a_{k'} :$$



$\rightarrow$ mapping of parameters:

$$\Gamma_0 = \Delta^2/\omega_c$$

$$U = 1 - \sqrt{2\alpha}$$

$$g = 2U - U^2 = 1 - 2\alpha$$

$$P(t) = \langle \sigma_z(t) \rangle = 1 - 2n(t)$$

$n(t)$: occupancy of level

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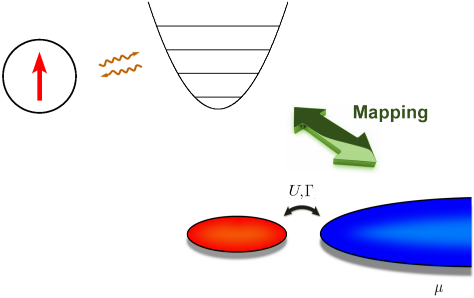

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- ADVANTAGE: $\alpha = 1/2 \rightarrow g = U = 0$ (Toulouse limit)

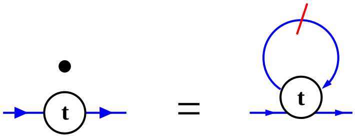
Leggett et al. '87, Weiss '12

Methods: FRG and RTRG

Functional RG: Keldysh Space

two-particle interaction: FRG

- cut-off Λ in Green functions:
  $G^0 \to G^{0,\Lambda}$ with $G^{0,\Lambda\to\infty} \to 0$ and $G^{0,\Lambda=0} \to G^0$
- hierarchy of flow equations for vertex functions
- truncate hierarchy after first order (only Σ flows)



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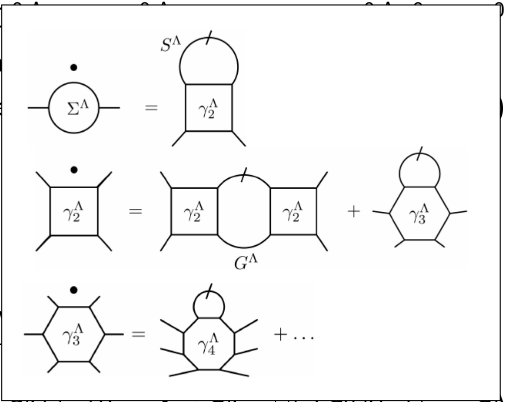

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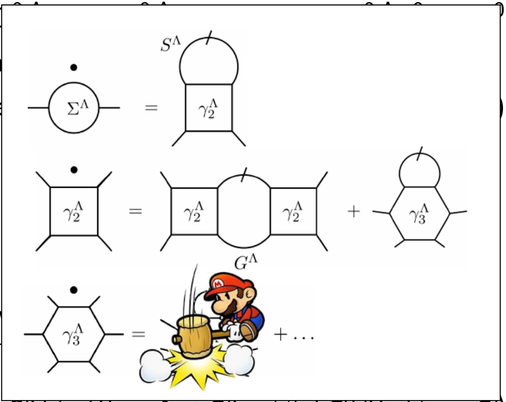

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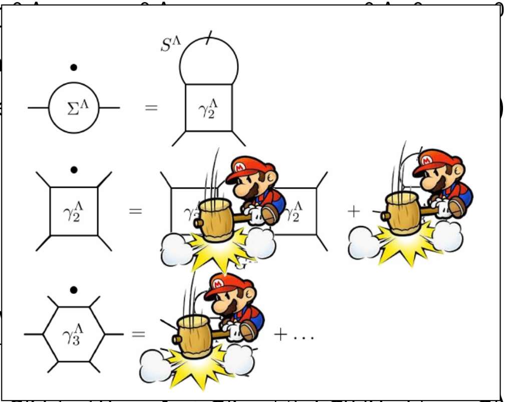

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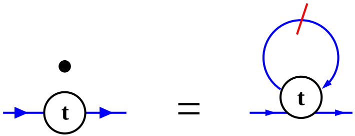

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Functional RG: Flexibility

steady driven state

periodic driving

transient dynamics

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$$G(t,t') = G(t-t')$$

$$\overset{\text{Fouriertransform}}{\Rightarrow} \quad G(\omega)$$

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Fouriertransform $\Rightarrow$ $G(\omega)$

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$t_0 \to -\infty$:

$$G(t, t') = G(t + T, t' + T)$$

Floquet theory $\Rightarrow$ $G_k(\omega)$

transient dynamics

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Real-Time RG: Liouville-Laplace Space

aim at reduced density matrix of local system

$$
\begin{aligned}
i\dot{\rho}(t) &= [H, \rho] = L\rho \\
\rho(t) &= e^{-iL(t-t_0)}\rho(t_0) \\
\rho_S(t) &= \mathrm{Tr}_{\mathrm{res}}\left[\rho(t)\right] \\
\rho_S(t) &= P(t)\rho_S(t_0) = \frac{i}{2\pi}\int_{-\infty+i0^+}^{\infty+i0^+} dE\, e^{-iEt}\,\Pi_1(E)\rho_S(t_0),
\end{aligned}
$$

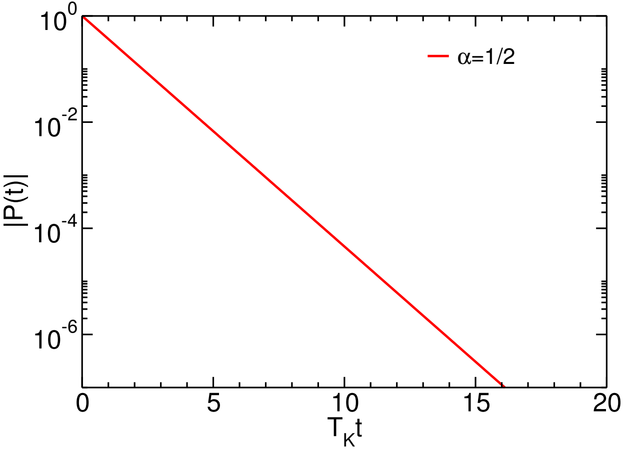
with $\Pi_1(E) = [E + i\Gamma_1(E)]^{-1}$ and Laplace variable E as flow parameter:

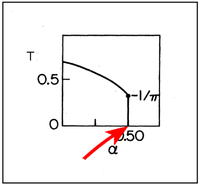
$$
\frac{d\Gamma_{1/2}(E)}{dE} = -(1-2\alpha)\,\Gamma_1(E)\,\Pi_{2/1}(E),
$$

with $\Pi_2(E) = [E + i\Gamma_2(E)/2]^{-1}$ and initially $\Gamma_n(i\omega_c) = \Delta^2/\omega_c$

Dynamics in the α-T-plane

Spin Relaxation Dynamics: $T = 0$





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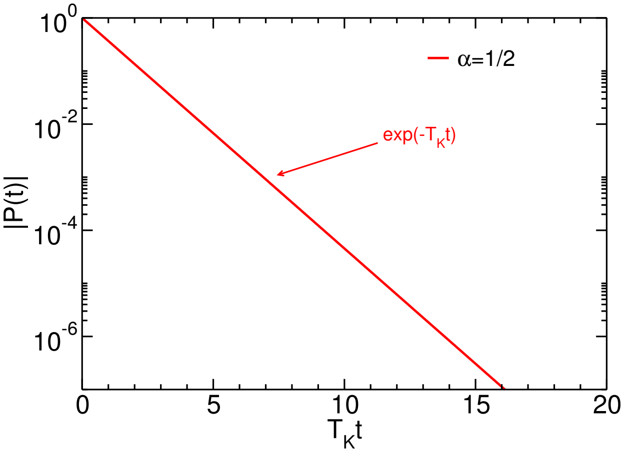

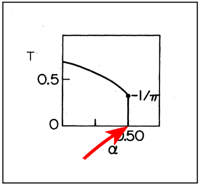

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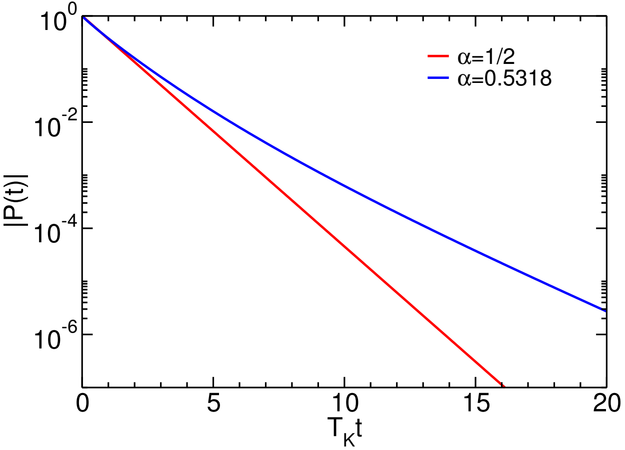

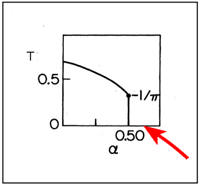

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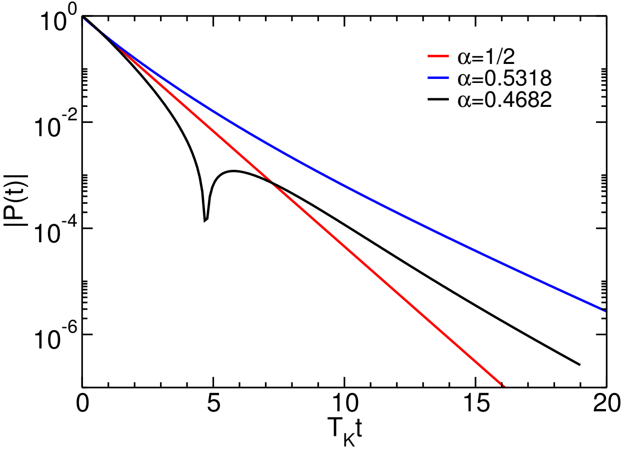

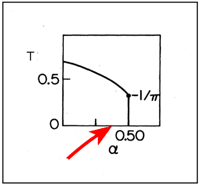

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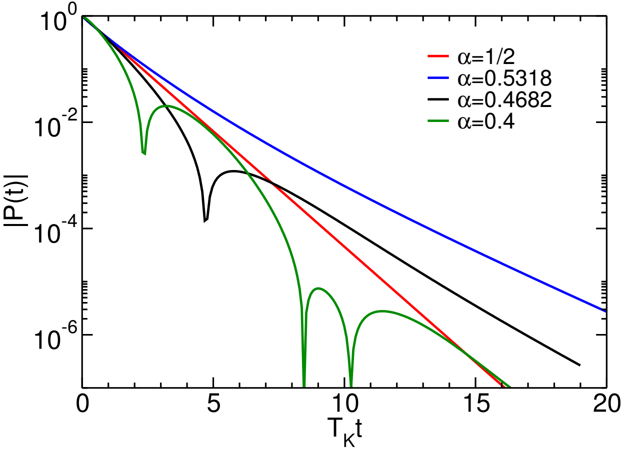

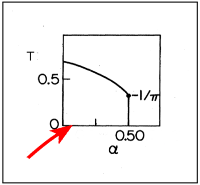

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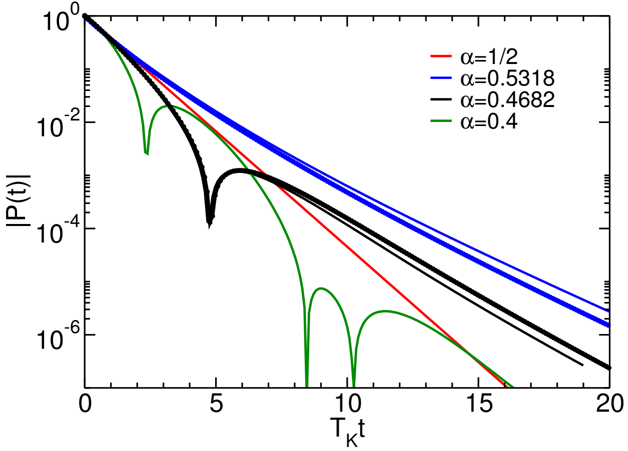

RTRG!

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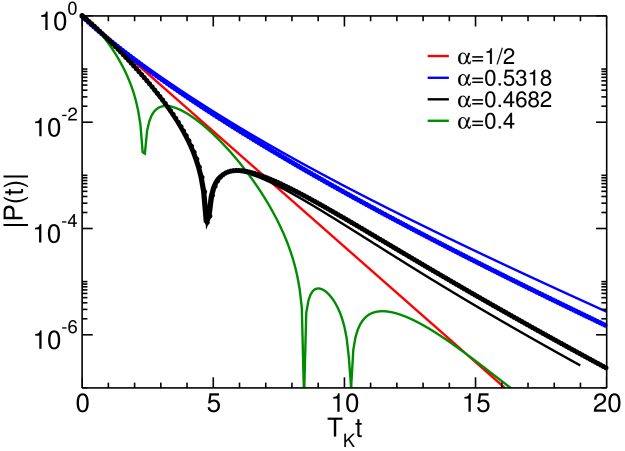
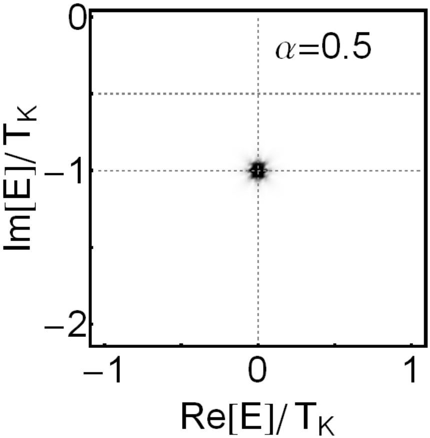


coherent regime: non-monotonic only for small-intermediate $T_K t$?

RTRG: Numerical Solution in Complex E-Plane

$$\left|\partial_{\mathrm{Im}E}\mathrm{Re}\left[\Pi_1\right] + \partial_{\mathrm{Re}E}\mathrm{Im}\left[\Pi_1\right]\right|$$



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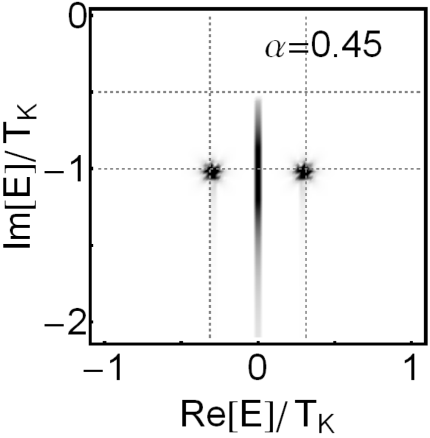

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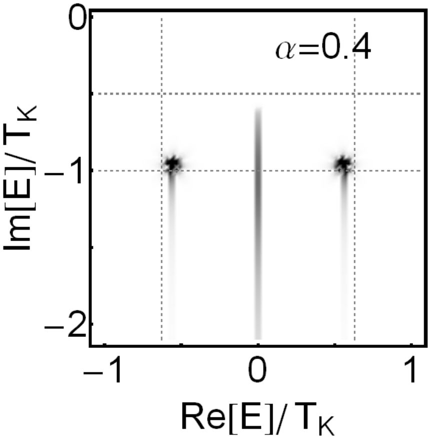

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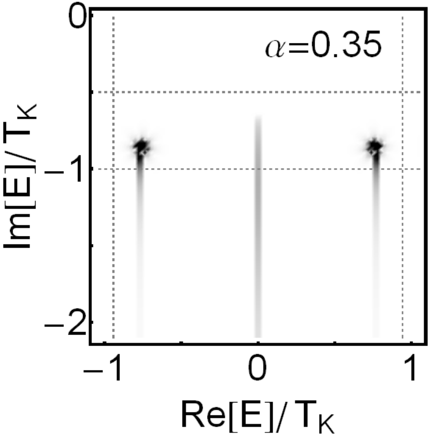

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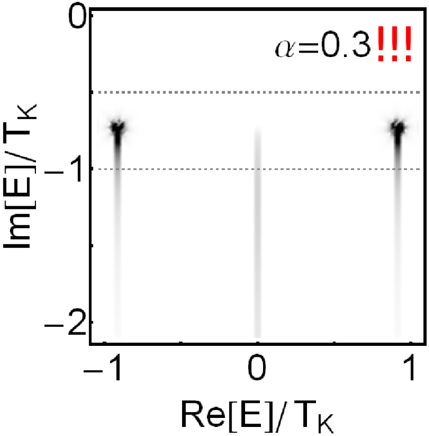

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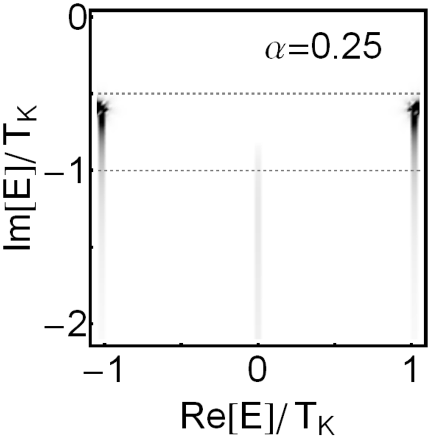

Intermezzo: Conclusion

- three distinct dynamical regimes
  - $1/2 < \alpha < 1$: incoherent
  - $\alpha_c < \alpha < 1/2$: partially coherent
  - $0 < \alpha < \alpha_c$: asymptotically coherent

with $\alpha_c \approx 0.3$

- still controlled?

RTRG for SBM (small α!) gives $\alpha_c \approx 0.36$

(Kashuba, Schoeller '13)

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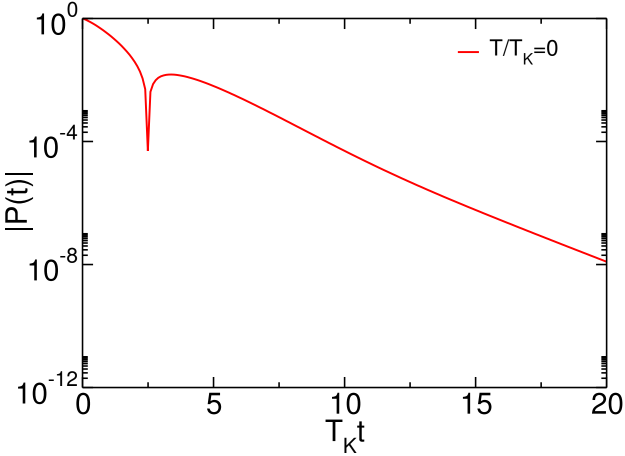
- FRG $\alpha = 0.4045$ (partially coherent)



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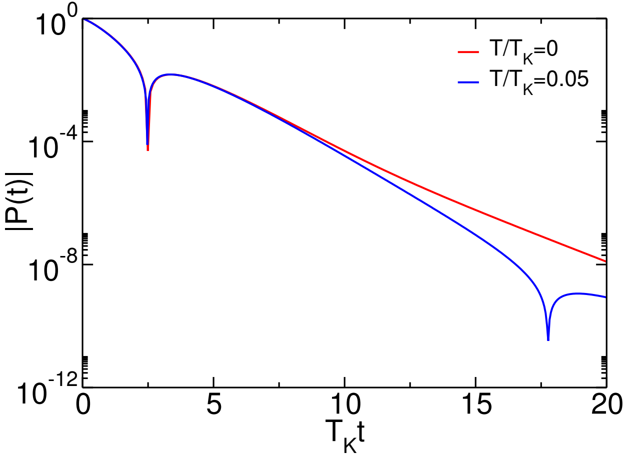

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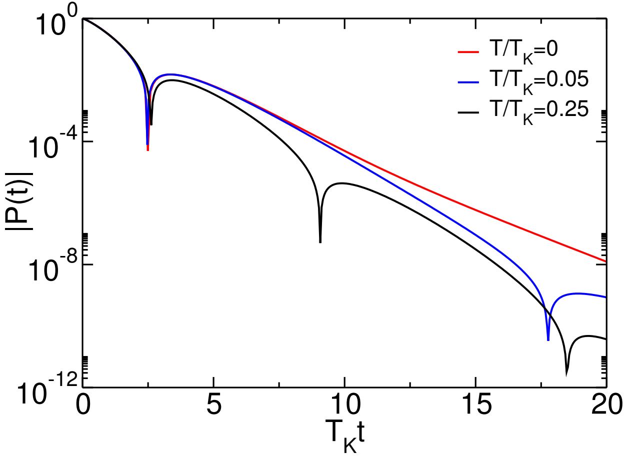

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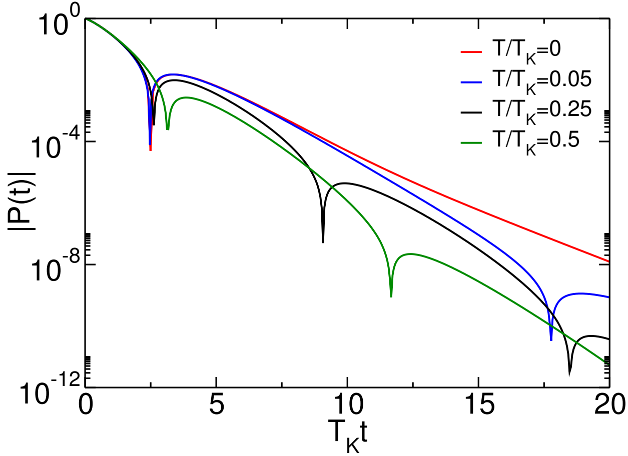

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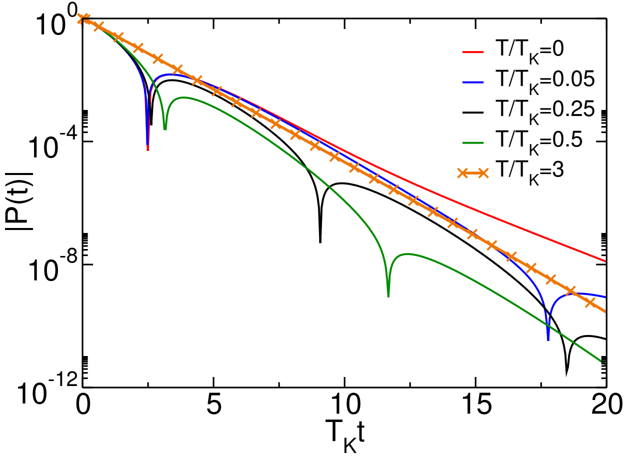
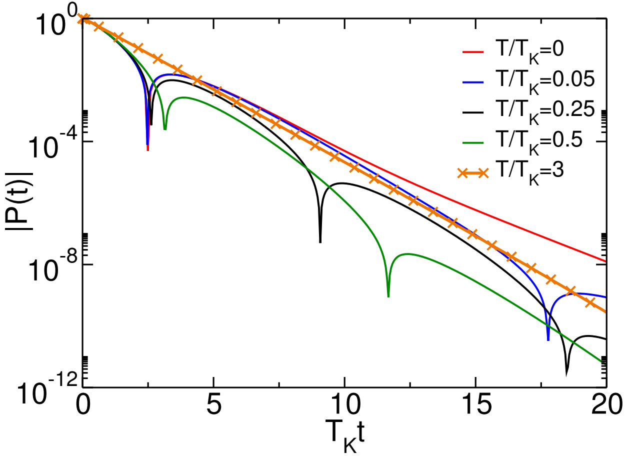


Spin Relaxation Dynamics: $T > 0$

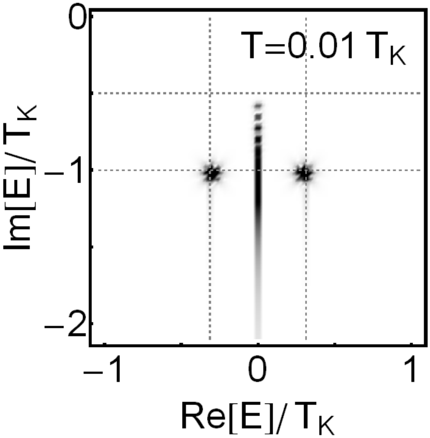
- FRG $\alpha = 0.4045$ (partially coherent)



- partially coherent $\rightarrow$ asymptotically coherent $\rightarrow$ incoherent?

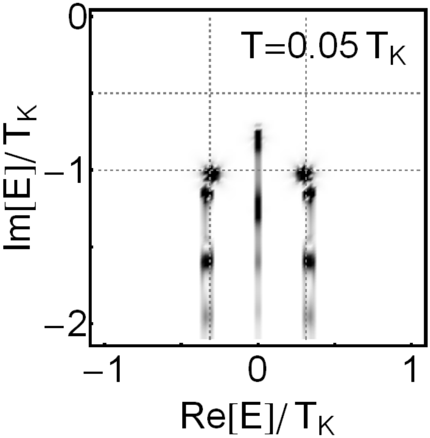
RTRG: Numerical Solution in Complex E-Plane $\alpha = 0.45$

$$|\partial_{\mathrm{Im}E}\mathrm{Re}\,[\Pi_1] + \partial_{\mathrm{Re}E}\mathrm{Im}\,[\Pi_1]|$$



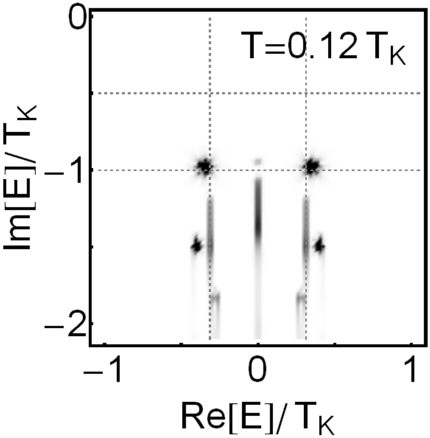
RTRG: Numerical Solution in Complex E-Plane $\alpha = 0.45$

$$\left|\partial_{\mathrm{Im}E}\mathrm{Re}\left[\Pi_1\right] + \partial_{\mathrm{Re}E}\mathrm{Im}\left[\Pi_1\right]\right|$$



RTRG: Numerical Solution in Complex E-Plane $\alpha = 0.45$

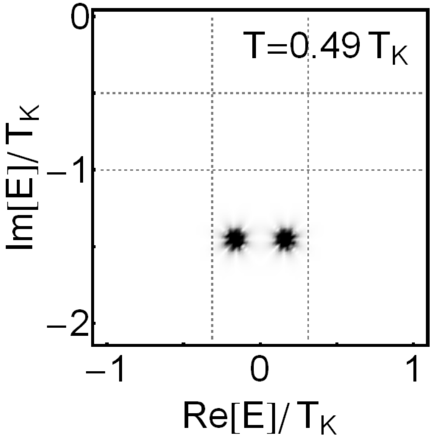
$$\left|\partial_{\mathrm{Im}E}\mathrm{Re}\left[\Pi_1\right] + \partial_{\mathrm{Re}E}\mathrm{Im}\left[\Pi_1\right]\right|$$



$T_{c_1}(\alpha)$!!!

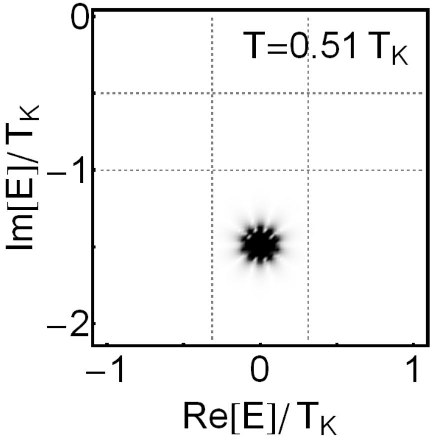
RTRG: Numerical Solution in Complex E-Plane $\alpha = 0.45$

$$|\partial_{\mathrm{Im}E}\mathrm{Re}\,[\Pi_1] + \partial_{\mathrm{Re}E}\mathrm{Im}\,[\Pi_1]|$$



RTRG: Numerical Solution in Complex E-Plane $\alpha = 0.45$

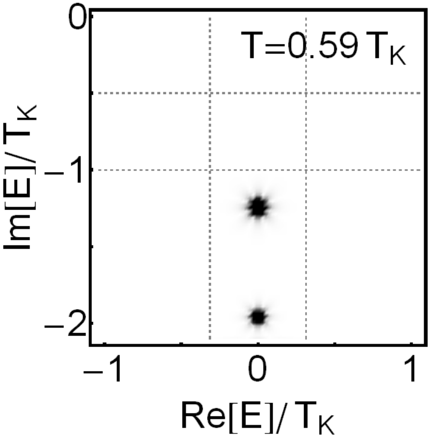
$$|\partial_{\mathrm{Im}E}\mathrm{Re}\,[\Pi_1] + \partial_{\mathrm{Re}E}\mathrm{Im}\,[\Pi_1]|$$



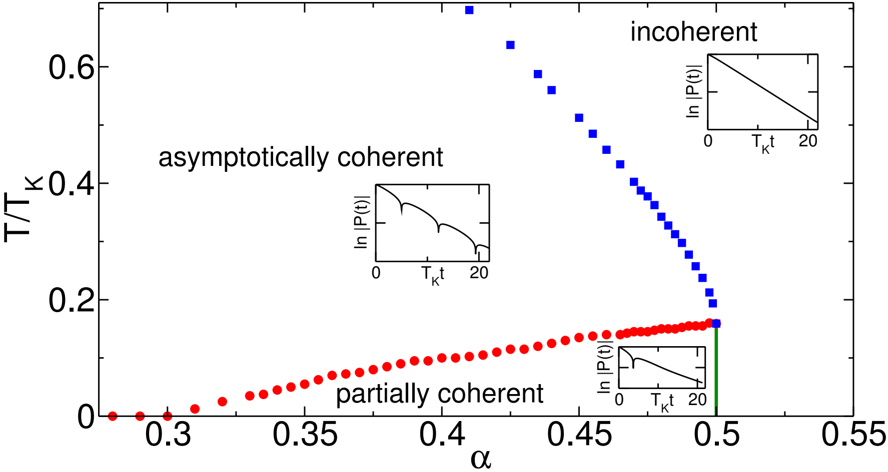
$T_{c_2}(\alpha)$!!!

RTRG: Numerical Solution in Complex E-Plane $\alpha = 0.45$

$$|\partial_{\mathrm{Im}E}\mathrm{Re}\,[\Pi_1] + \partial_{\mathrm{Re}E}\mathrm{Im}\,[\Pi_1]|$$



Conclusion: Dynamical Regimes



$T = 0$: PRL **110**, 100405 (2013)
$T > 0$: PRB **88**, 241110(R) (2013)

Summary

dynamics in α-T-plane

- three distinct dynamical regimes
- increase T : transition from partially to asymptotically coherent
- 'coherence from elevated temperature'

$T = 0$: Phys. Rev. Lett. **110**, 100405 (2013)
Quench: Phys. Rev. B **88**, 165133 (2013)
$T > 0$: Phys. Rev. B **88**, 241110(R) (2013)

avenues of future research

- periodic driving (see Eissing et al. arXiv:1508.01325)
- current noise

- ...