

Strong correlation effects in topological quantum phase transitions

Adriano Amaricci

NGSCES-2015, Trogir, Sept 2015



...when Correlation meets Topology

Correlated crew...



G. Sangiovanni



M. Capone

Topological crew...

J. Budich



B. Trauzettel



The phases of the matter are usually classified according to Ginzburg-Landau (*symmetry breaking*) theory...

Magnetism



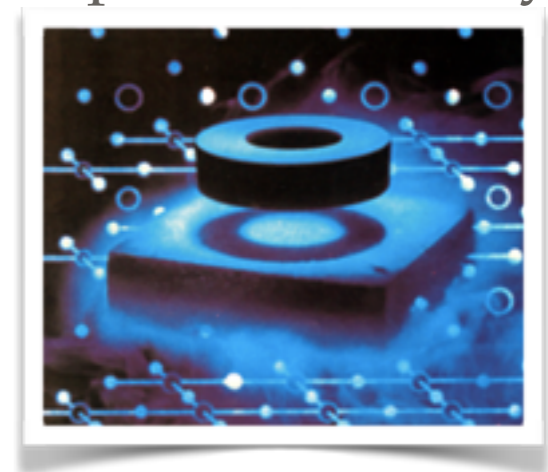
magnetization M

Liquid-gas



density difference $n(L)-n(G)$

“Superconductivity”



pair amplitude ψ

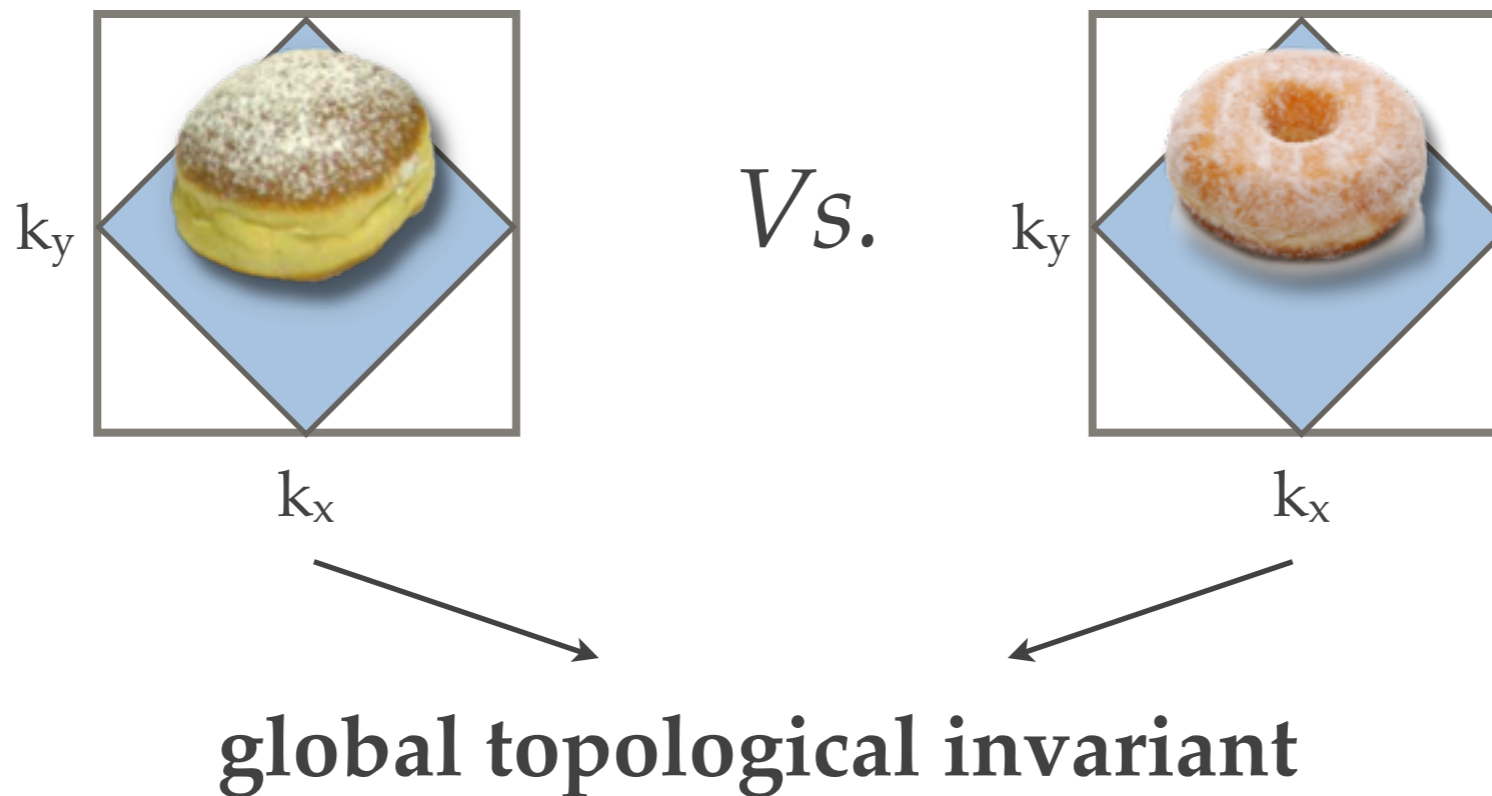
local order parameter

Experimental detectability!

Last years witnessed the emergence of the concept of

TOPOLOGICAL INSULATORS

quantum materials which elude the conventional G-L paradigm...



hard to relate with the bulk observables

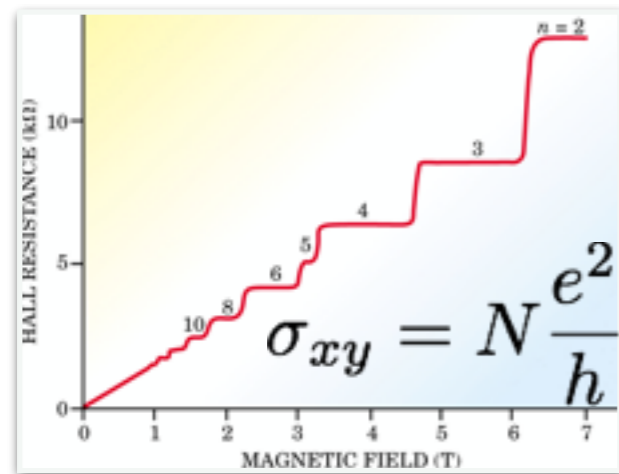
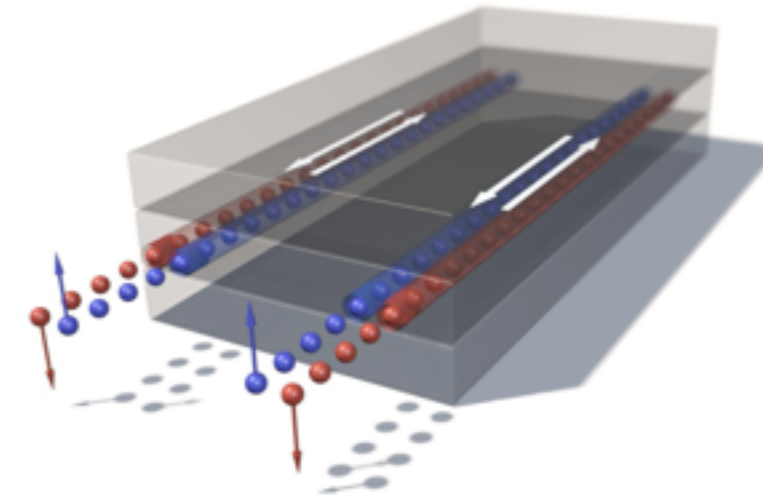
TI + geometric detour

what is a TI?

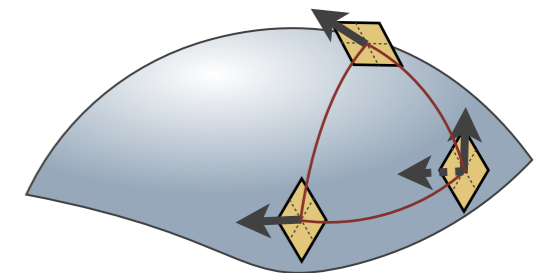
a bulk (band) insulator + with *gapless* edge modes.

Dirac semi-metal + *Spin-Orbit Coupling*

(Haldane PRL88, Kane,Mele PRL05)



States classified in terms of the *Topological Properties* of the Hilbert space of Bloch functions:



Berry connection: $\mathcal{A}_n = \langle u_n(k) | -i \nabla_k | u_n(k) \rangle$ parallel transport of Bloch states

Berry curvature: $\mathcal{F}_n = \nabla_k \times \mathcal{A}_n$ Gauss-Bonnet: $N_n = \frac{1}{2\pi} \int_{BZ} d^2k \mathcal{F}_n$

N is the Chern number \rightarrow Hall cond., # edge states, Z_2 invariant

BHZ model *(quantum spin Hall insulator)*

Initial focus on graphene but small SOC ($gap \sim 10^{-3} meV$)

(Kane, Mele PRL 2005)

Idea: look for systems with a larger SOC.

(Bernevig et al Science 2006)
(Konig et al Science 2007)

BHZ model:

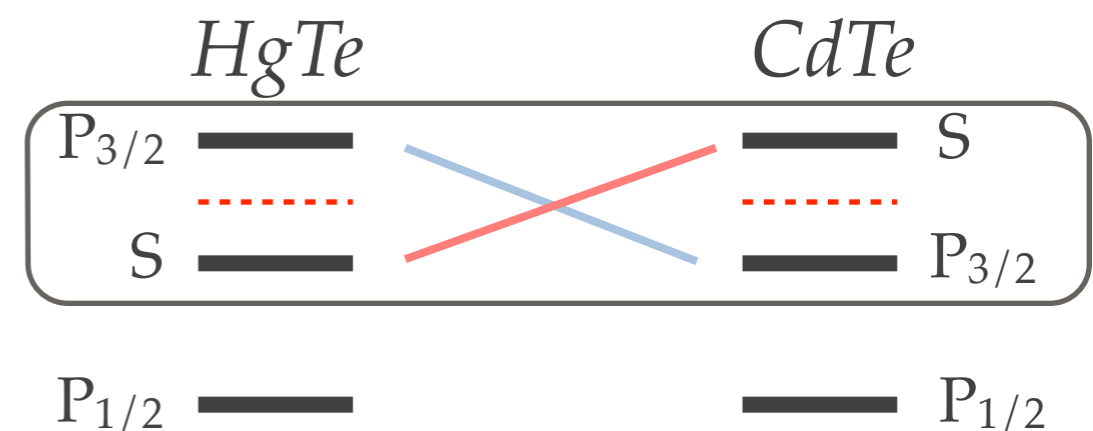
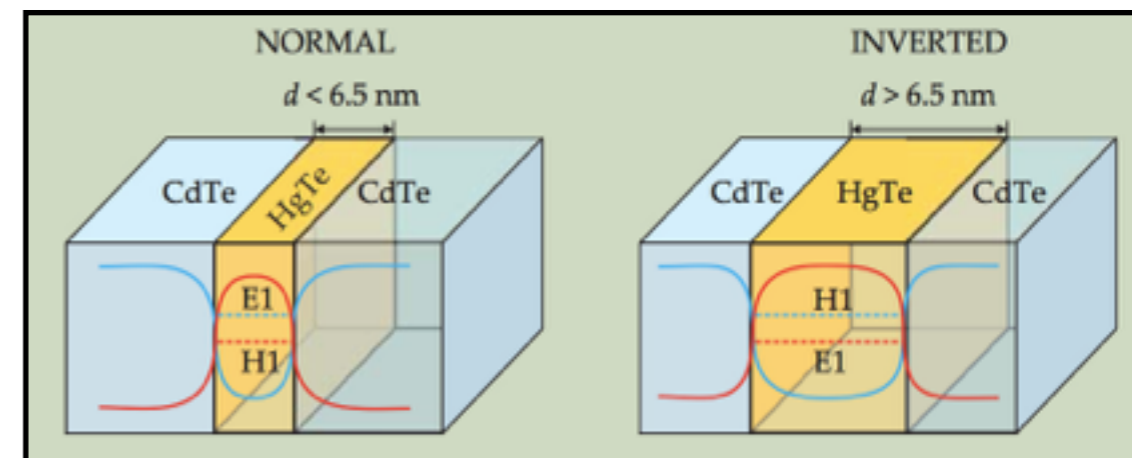
2 QHI + Time Reversal Symmetry.

$$H = \begin{pmatrix} \mathbf{h}(\mathbf{k}) \uparrow & \mathbf{0} \\ \mathbf{0} & \mathbf{h}^*(-\mathbf{k}) \downarrow \end{pmatrix} \text{ Spin structure}$$

$$\mathbf{h}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau} \text{ Orbital pseudo-spin structure}$$

$$\mathbf{d}(\mathbf{k}) = \begin{pmatrix} \lambda \sin k_x \\ \lambda \sin k_y \\ M - \cos k_x - \cos k_y \end{pmatrix}$$

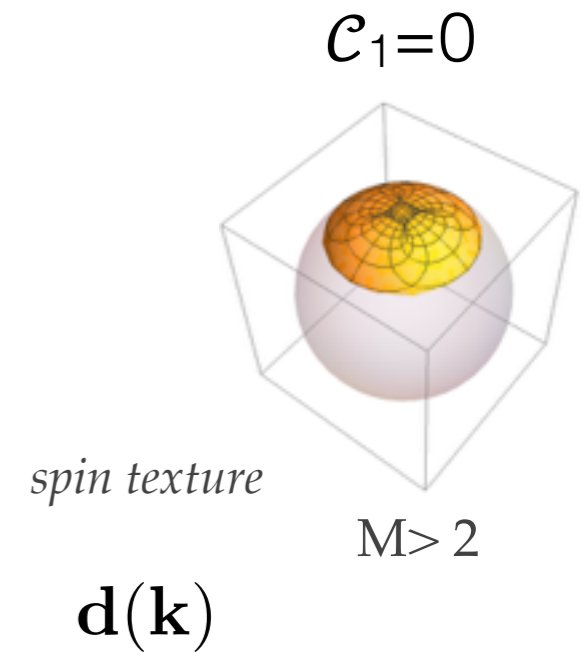
CdTe/HgTe quantum wells.



Topological QPT

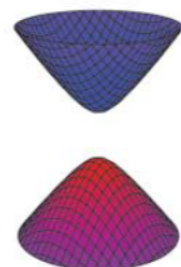
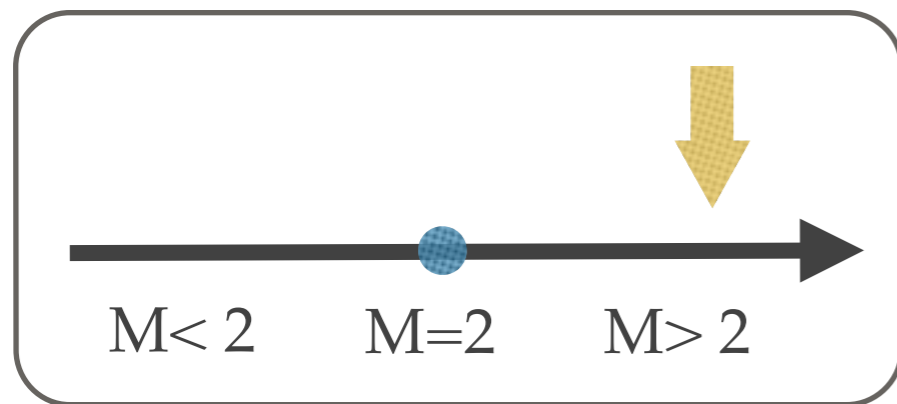
BHZ description of topological transition:

$$\mathbf{h}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau}$$

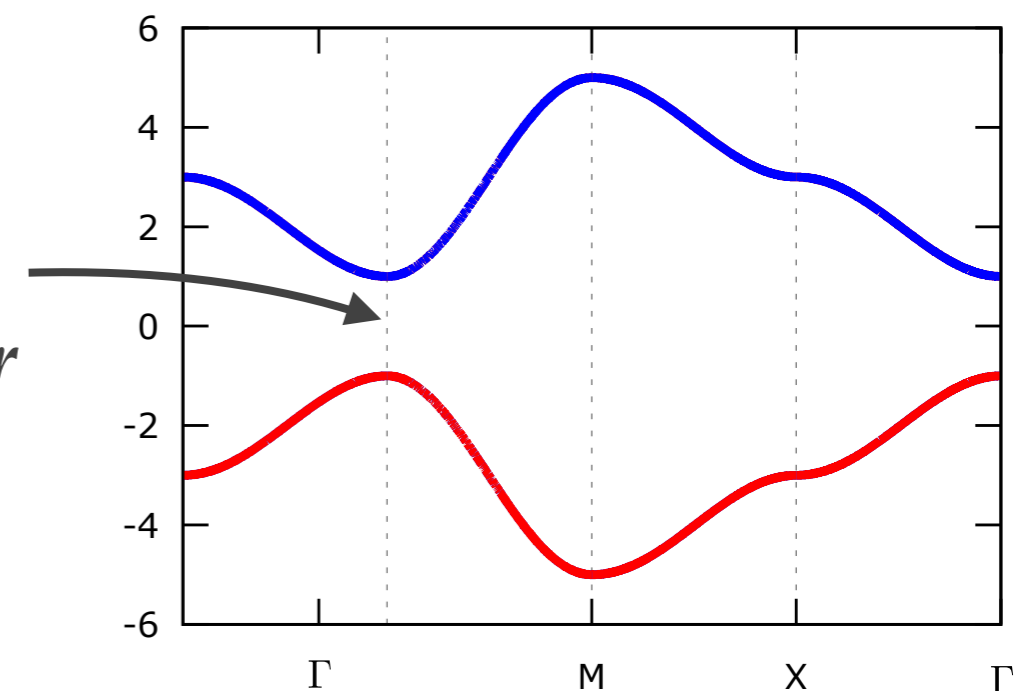


Continuous Topological Quantum Phase Transition

band structure evolves smoothly with control parameters...



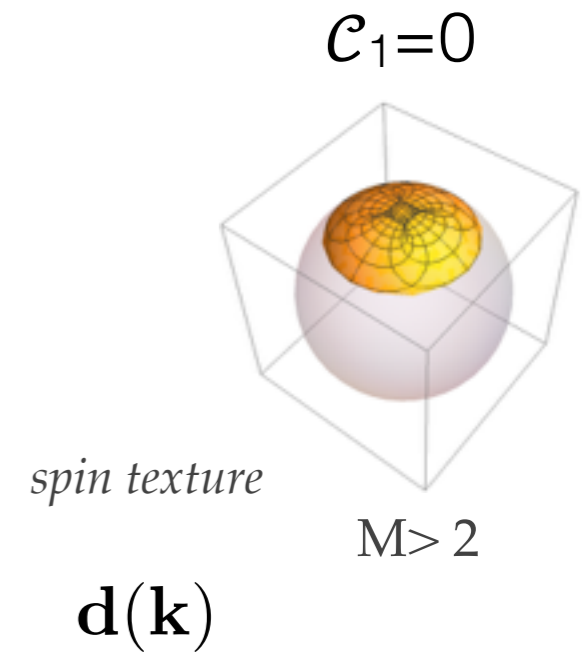
*trivial
band insulator*



Topological QPT

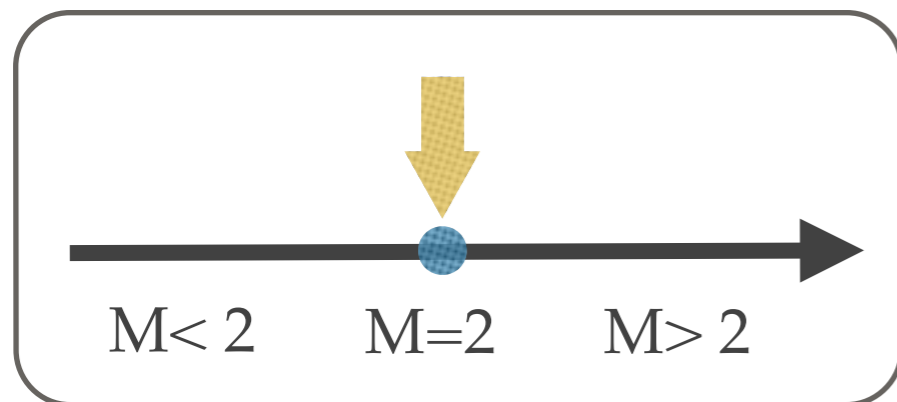
BHZ description of topological transition:

$$\mathbf{h}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau}$$

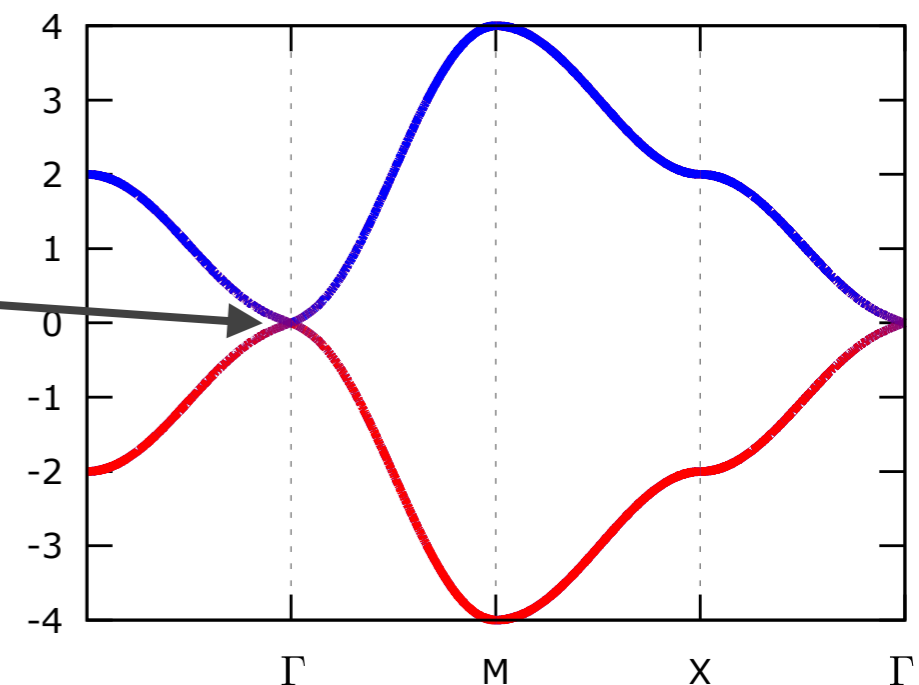


Continuous Topological Quantum Phase Transition

band structure evolves smoothly with control parameters...



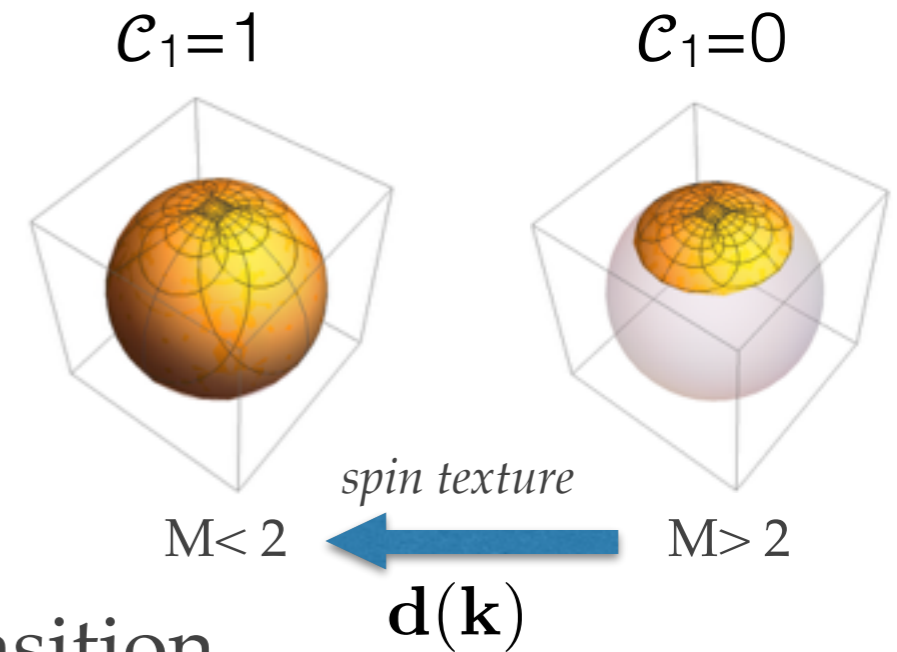
Dirac cone semi-metal



Topological QPT

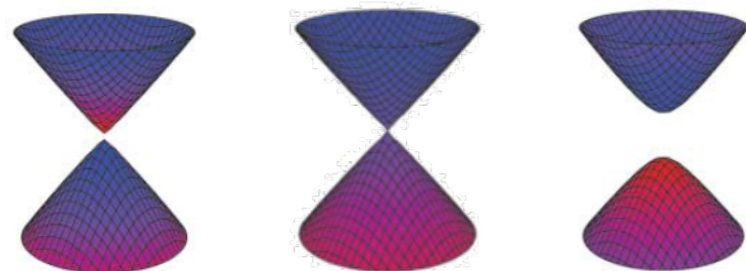
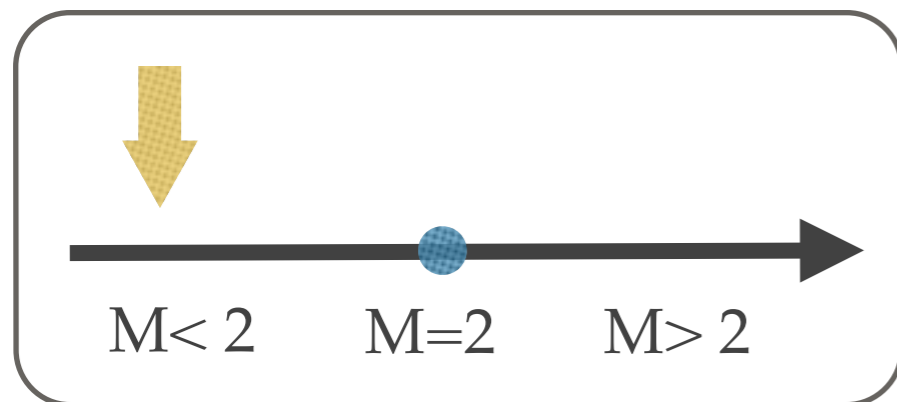
BHZ description of topological transition:

$$\mathbf{h}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau}$$

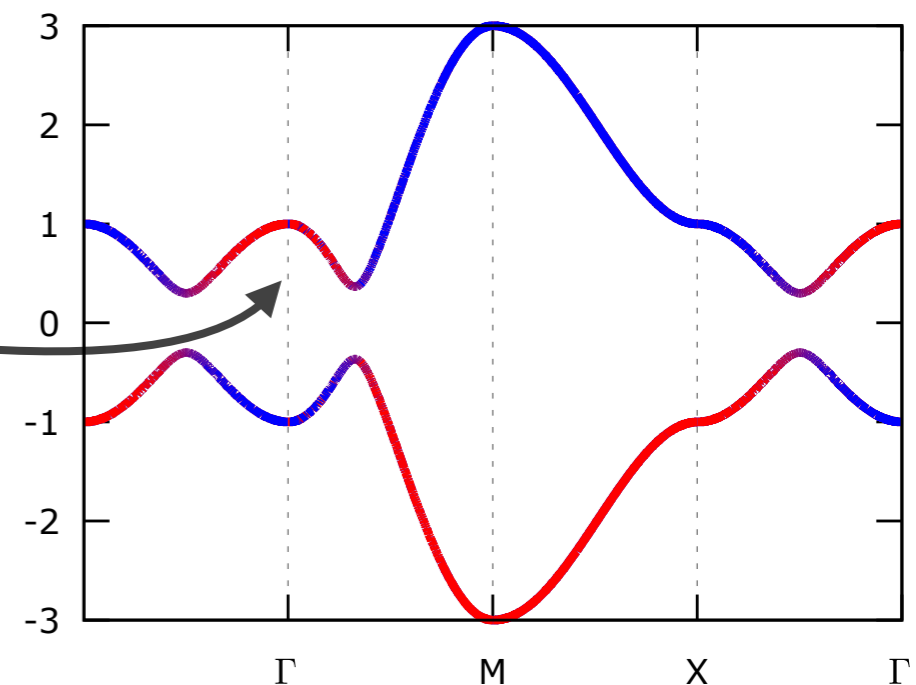


Continuous Topological Quantum Phase Transition

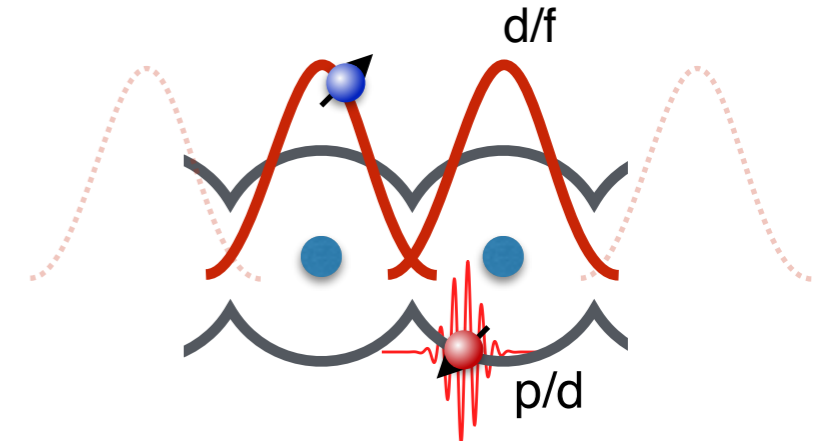
band structure evolves smoothly with control parameters...



band inversion
QSH insulator



Quest for larger SOC...



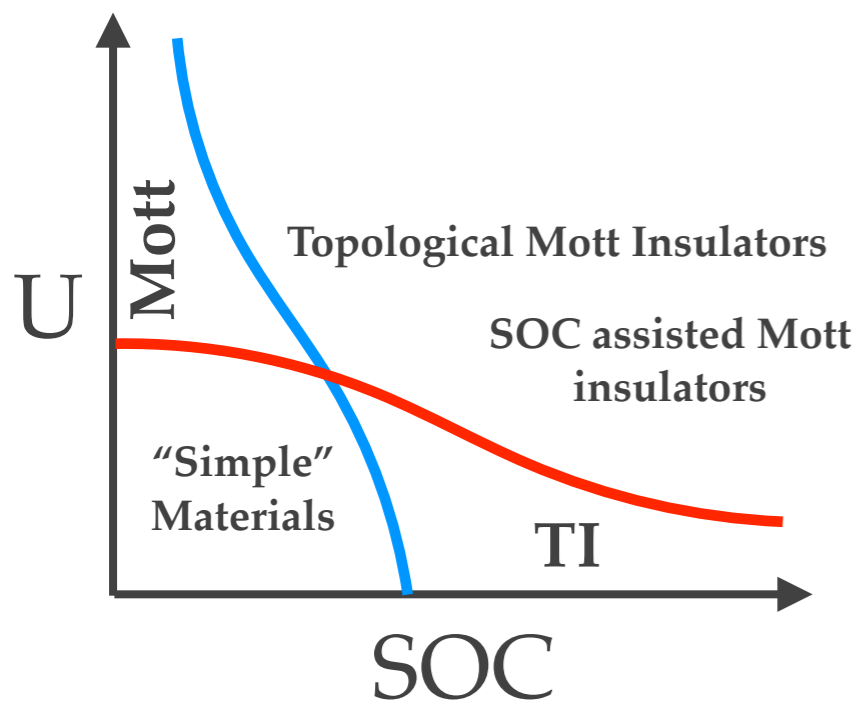
...heavy elements compounds (5d/4f)...

e.g. hexaborides Sm/PuB_6 ,

Dzero et al. PRL 2010

Ir-based pyrochlores: e.g. $\text{Sr}_2\text{Ir}_2\text{O}_7$, etc..

D. Pesin, L. Balents, NP 2010



from L. Balents website



...large electronic interaction,

SOC strongly reacts to it

Hohenadler, Assad. Journal of Phys. 2013

What is the fate of the TQPT?

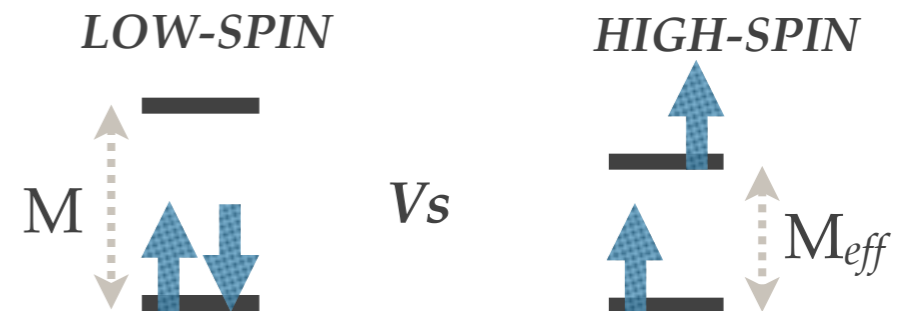
BHZ + Hubbard

Let's take into account multi-orbital **interactions**:

$$H = \begin{pmatrix} \mathbf{h}(\mathbf{k}) & \mathbf{0} \\ \mathbf{0} & \mathbf{h}^*(-\mathbf{k}) \end{pmatrix} + (U - J) \frac{N_i(N_i - 1)}{2} - J \left(\frac{N_i^2}{4} + S_{zi}^2 - 2T_{zi}^2 \right)$$

density *spin* *orbital*

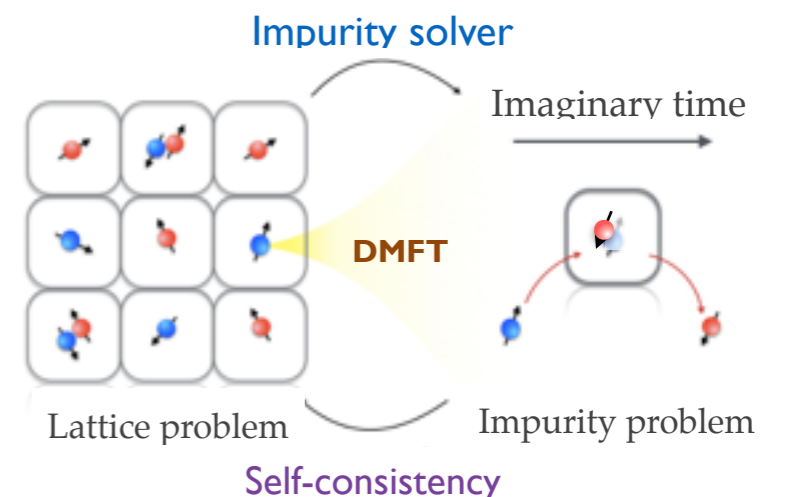
Hund's coupling: *maximize* $\langle S_z^2 \rangle$
 minimize $\langle T_z^2 \rangle$



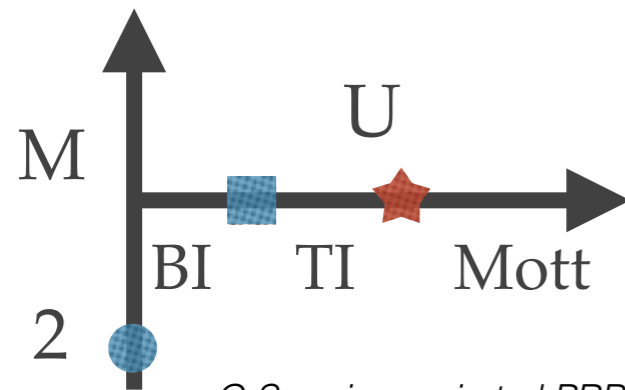
Dynamical Mean-Field Theory at T=0.

Dynamical effects of interaction are encoded in the self-energy function.

$$\hat{\Sigma}(\omega) = \text{Re}\Sigma(\omega)\sigma_0 \otimes \tau_z + \text{Im}\Sigma(\omega)\sigma_0 \otimes \tau_0$$



Interaction driven transitions



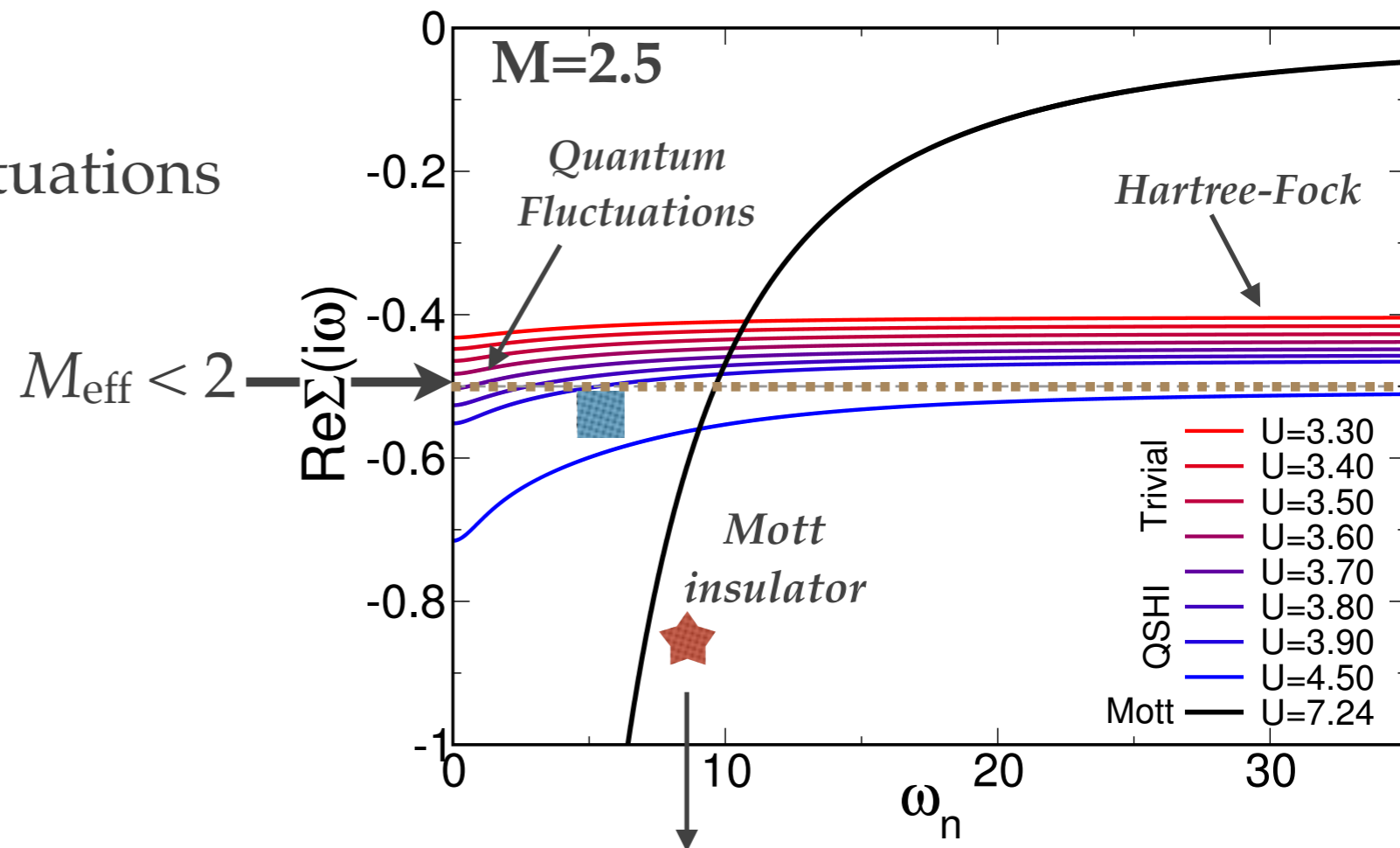
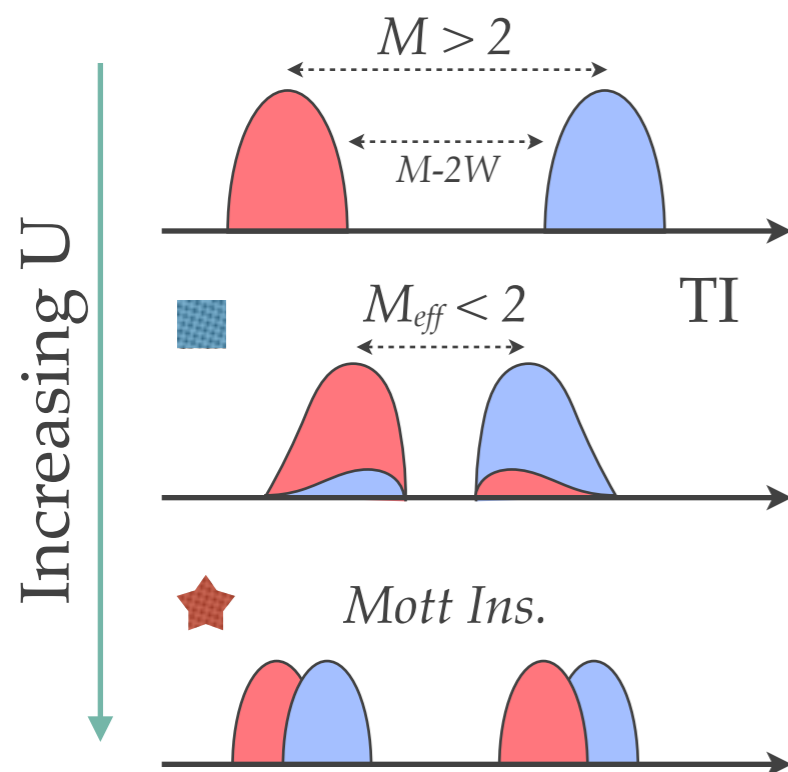
G.Sangiovanni et al PRB 2012

AA et al PRL 2015

- Effective reduction of Mass term

$$M_{\text{eff}} = M + \text{Tr}[\tau_z \hat{\Sigma}(0)]/2$$

(Interaction) Quantum Fluctuations driven TQPT



- Increasing U drives the system into a **Mott phase** with one electrons per orbital.

Phase-diagram

Phase diagram (*flipped view*): $1/M$ vs. $1/U$. *Liquid-Gas analogy.*

Color code reflects correlation degree: $\text{Tr}[\tau_z \hat{\Sigma}_{\text{HF}} - \tau_z \hat{\Sigma}(0)]/2$

Weak coupling: Hartree-Fock like. Continuous as the $U=0$ case.

Strong coupling:

1st order transition!

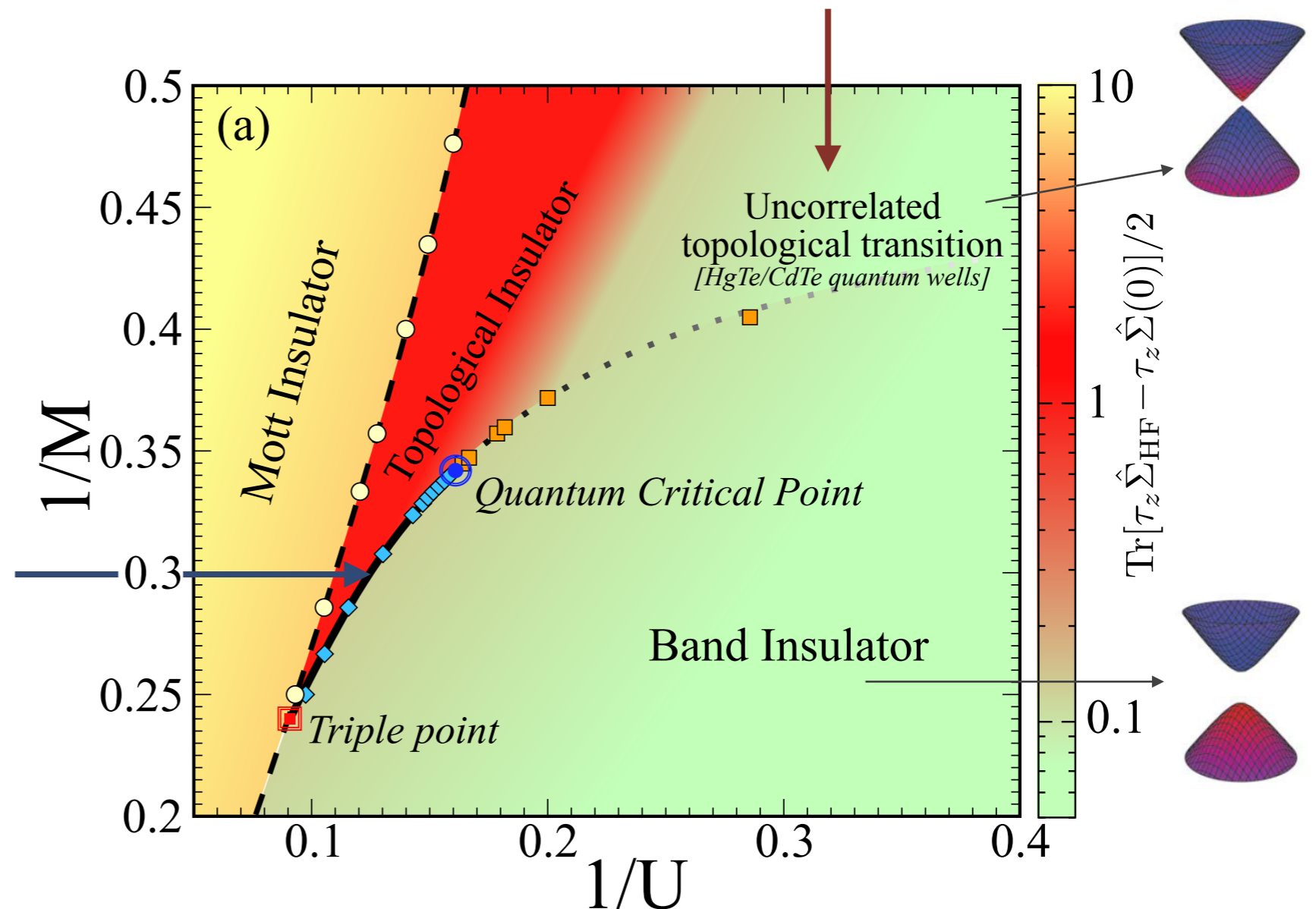
many-body character

strongly correlated

different ground states

AA et al PRL 2015

recently re-discovered in
Rau, Goswami et al arXiv: 1507.00722



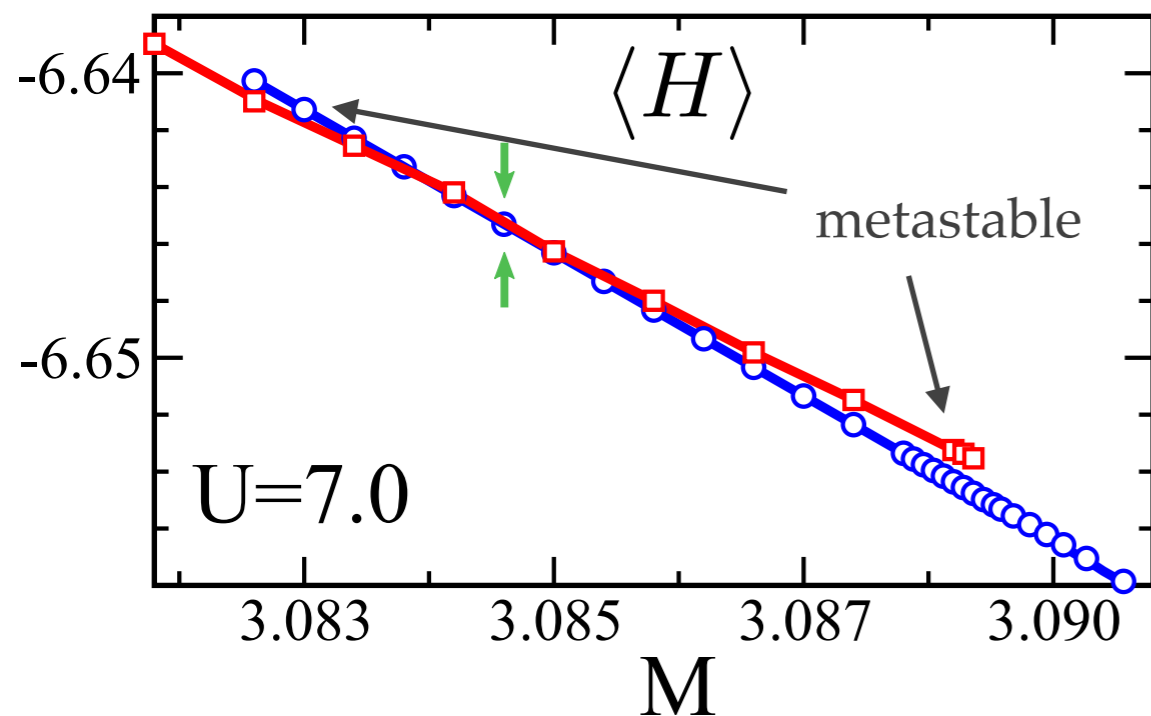
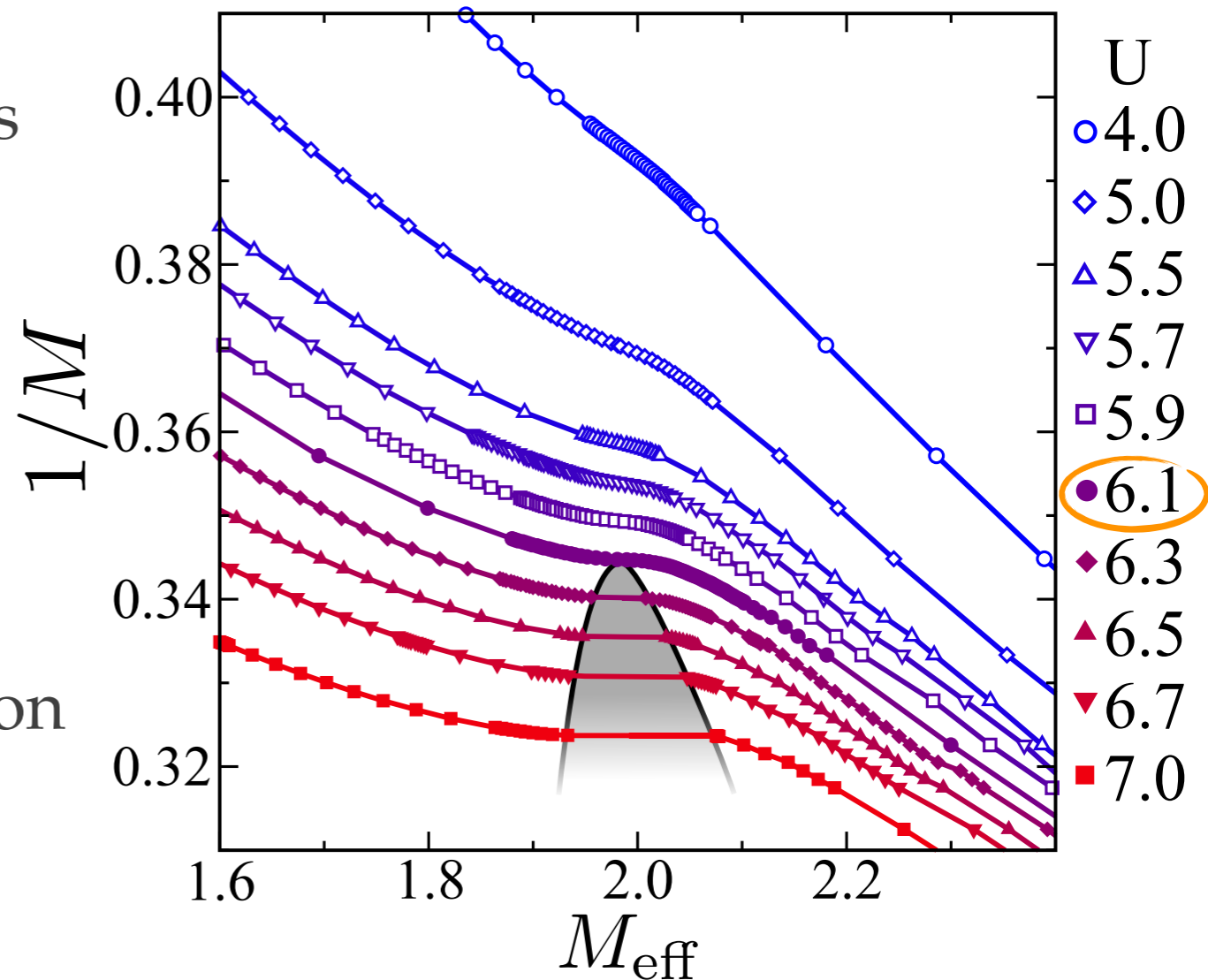
Topological QPT

A clear picture from the *iso-U* curves

- For $U < U_c$ continuous transition
- For $U = U_c$ critical behavior
- For $U > U_c$ discontinuous transition

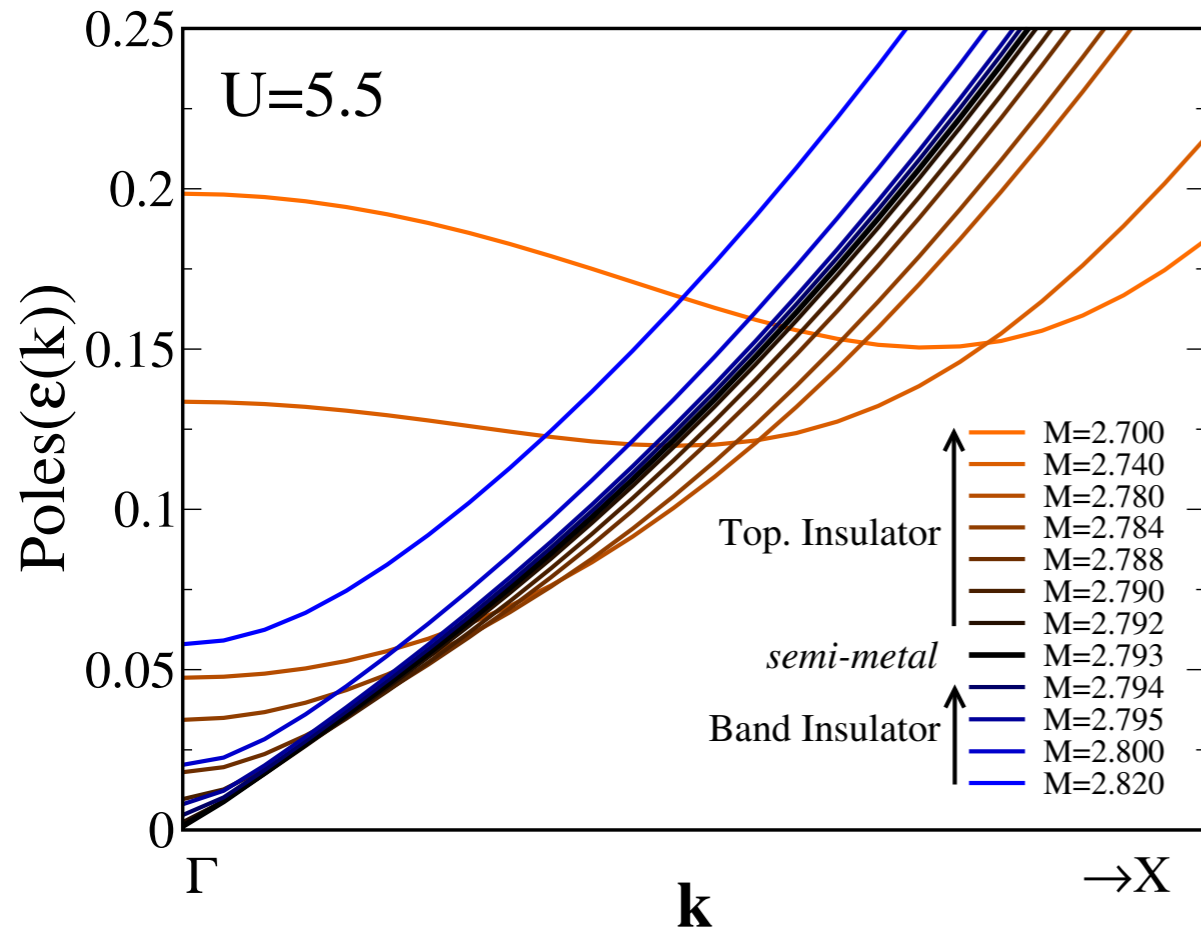
$$\Delta M_{\text{eff}} = M_{\text{eff}}(BI) - M_{\text{eff}}(QSH)$$

is the order parameter of the transition



Signature of the 1st-order character:
Hysteretic behavior of the total energy,
metastable states, coexistence.

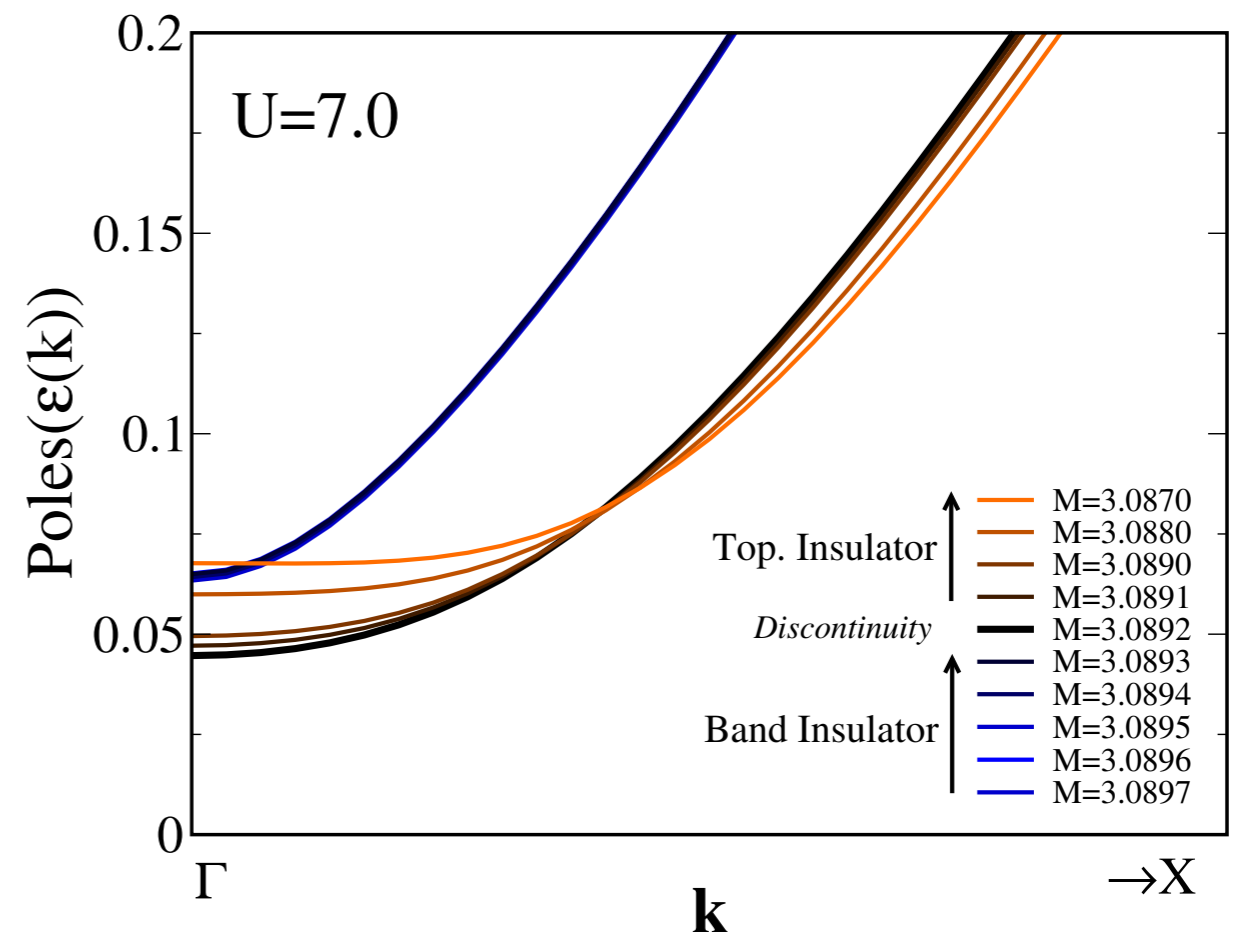
Absence of gap closure



$U < U_c$: the transition to a topological state occurs in the usual band-gap closing way.

Dirac cone (*gapless*) at the transition.

$U > U_c$:
 No gap-closing
 No suppression of any symmetries
 connected to the topological ordering.



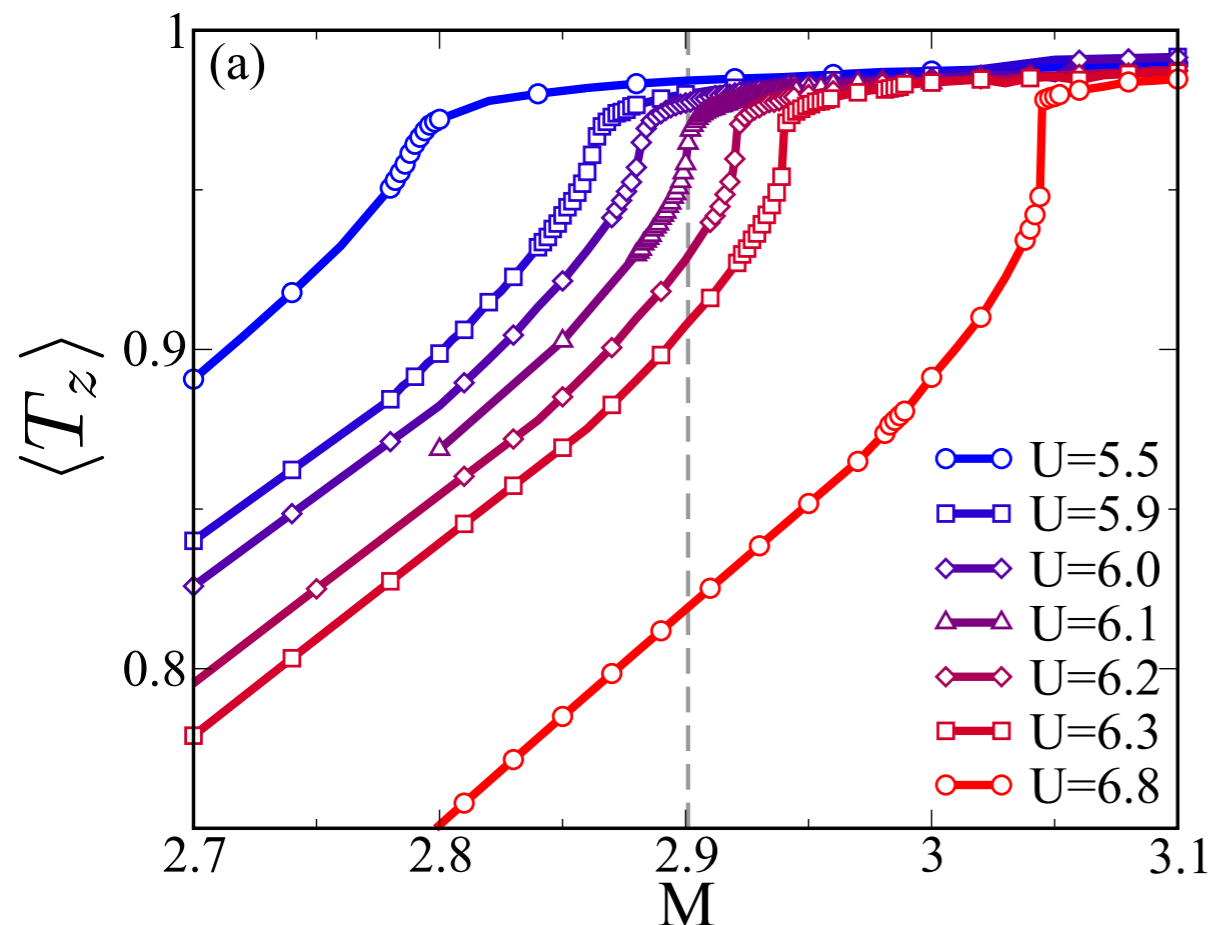
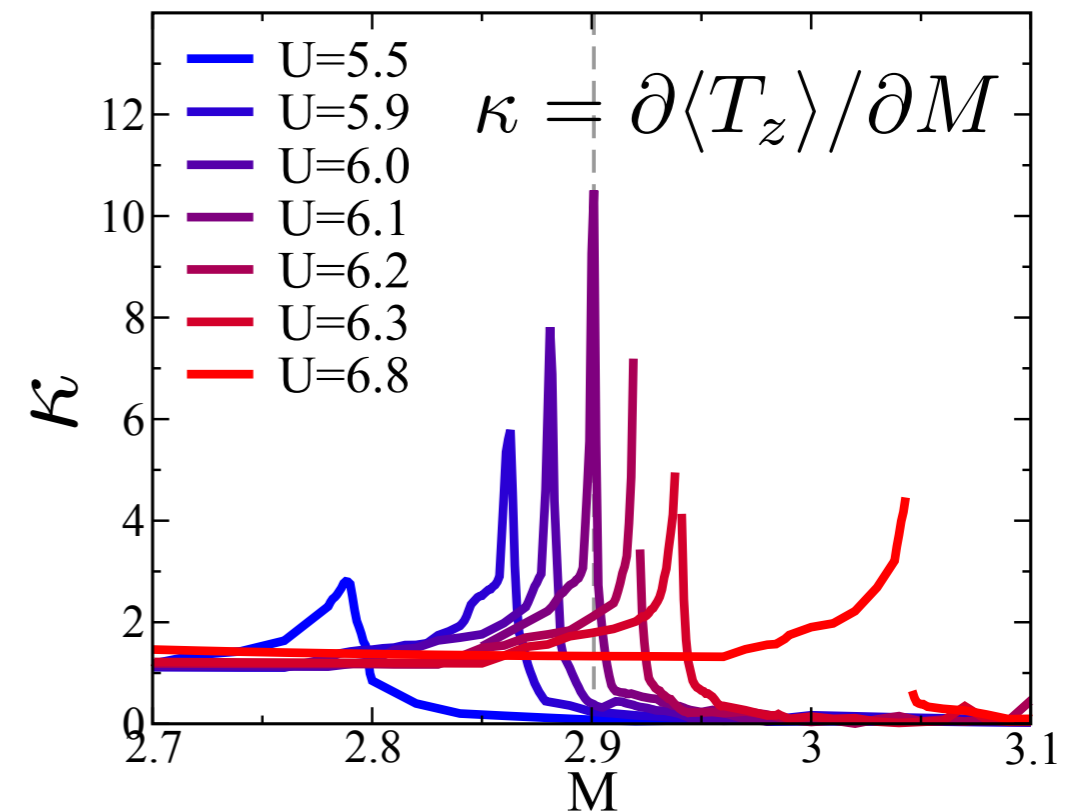
Experimental signatures.

Like liquid and gas become distinguishable at low-T only...

Clearcut experimental distinction between trivial and non-trivial phase.

Instability in the orbital sector

Diverging orbital compressibility at $U=U_c$



Critical orbital polarization T_z

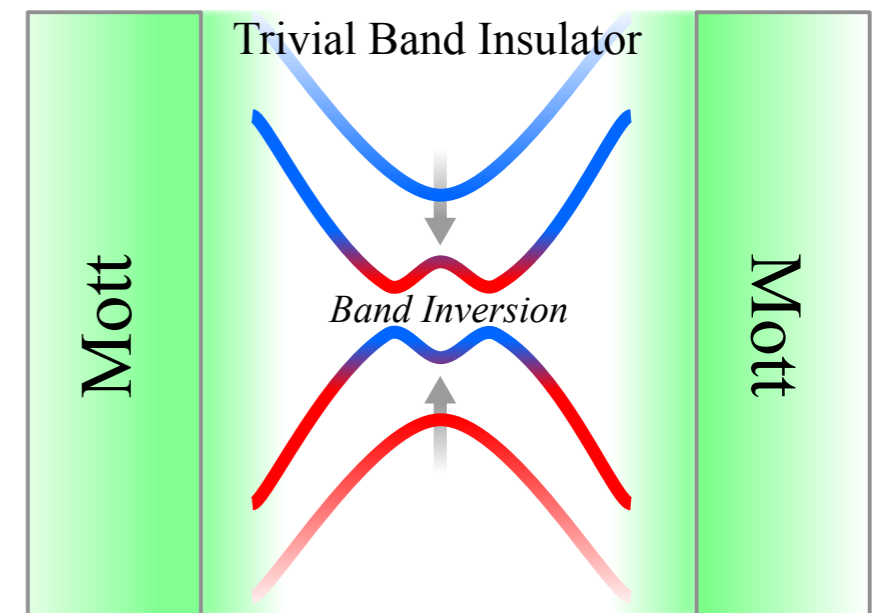
diverging fluctuations at the QCP

Experimental accessible quantities marking the topological phase-transition.

- Interaction driven QSHI states.
- Emergent thermodynamic character: 1st order transition.
- Topological QPT with NO gap closing & no-symmetry breaking!
- Experimental signatures of TQPT in correlated materials.

Outlook & Works in Progress...

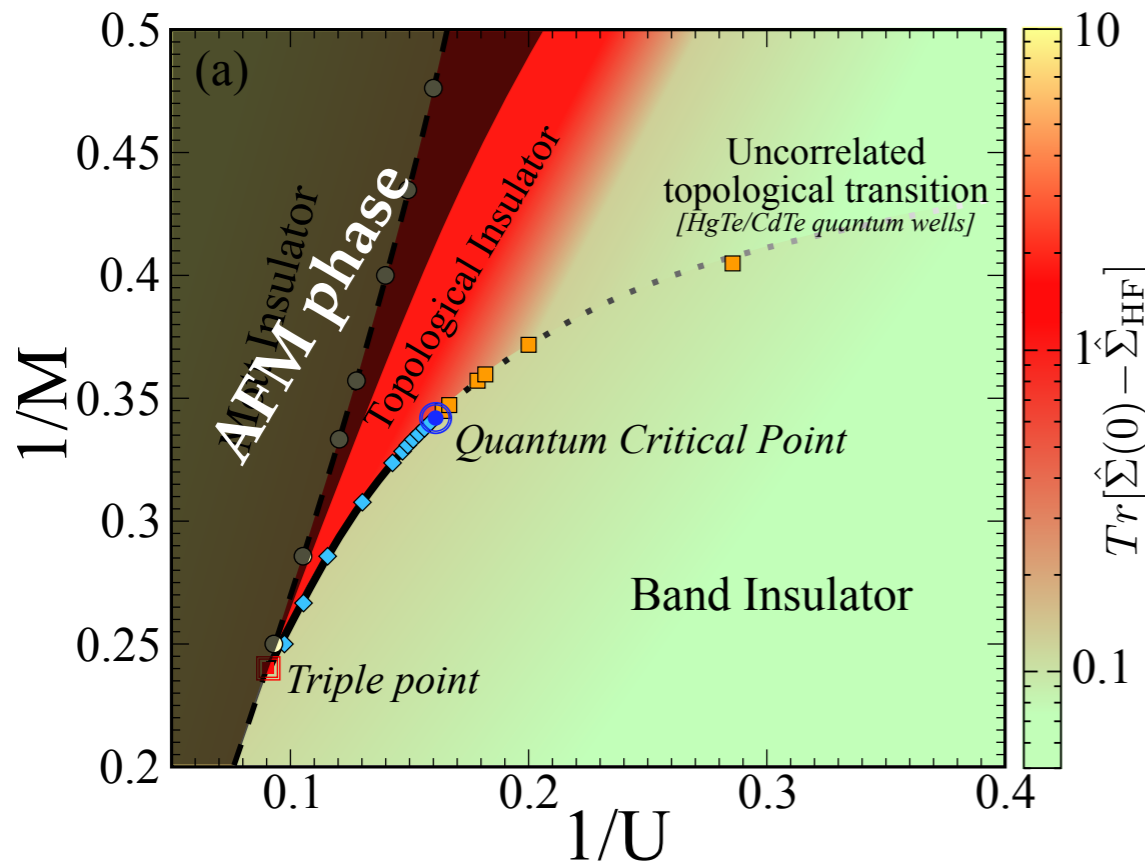
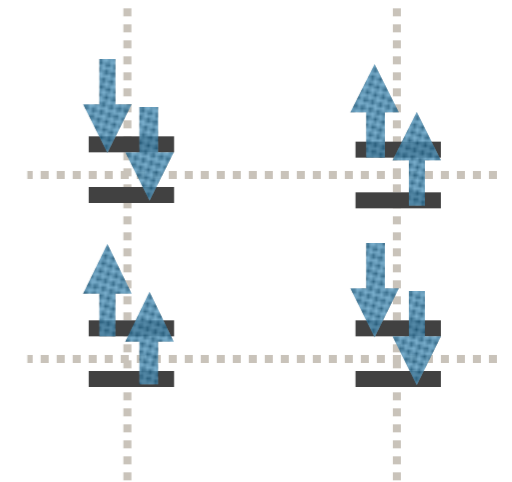
- Real materials with ab-initio + DMFT.
- Engineer a correlated QSH state.
- **Breaking SU(2)! Interacting Weyl Semi-Metals.**



Anti-ferromagnetic order

Large local moments: unstable to magnetic order.

AFM “g-type” phase in proximity of the Mott Insulator.

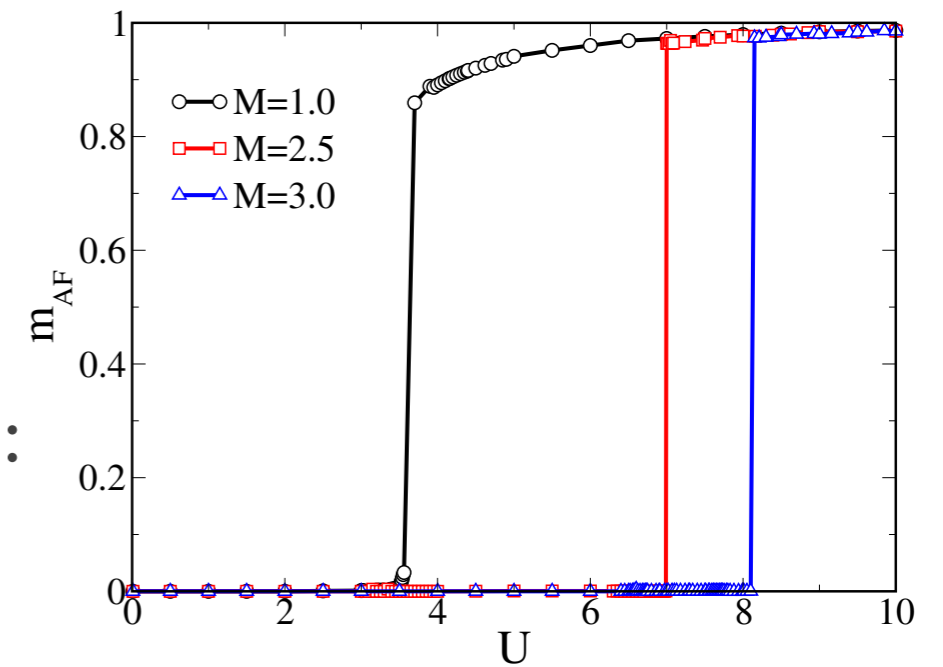


Ordered phase competes with TI.

Correlated-QSHI robust against AFM.

No traces of non-trivial AFM.

At $T=0$ AF magnetisation shows a large jump:
1st order transition.

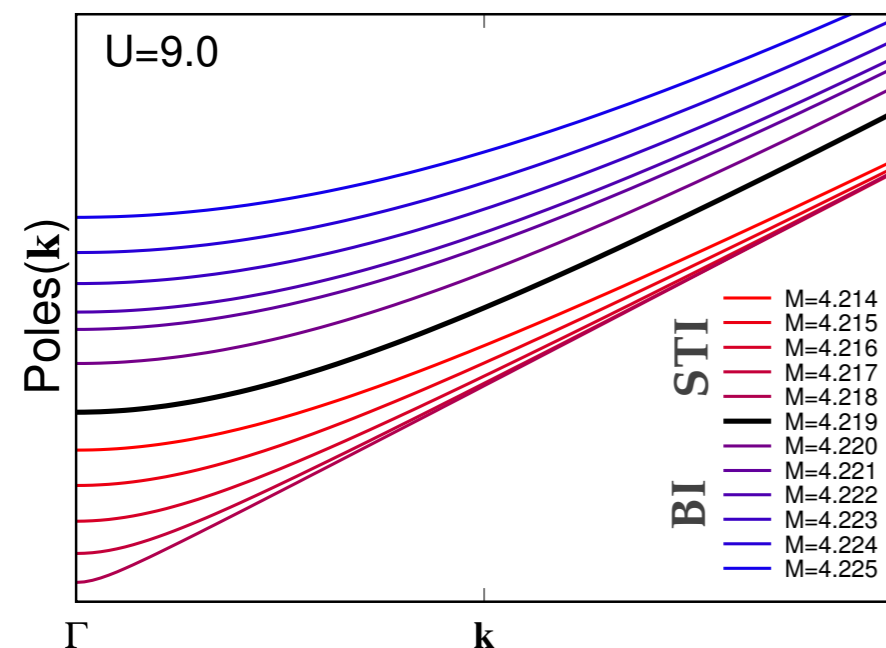
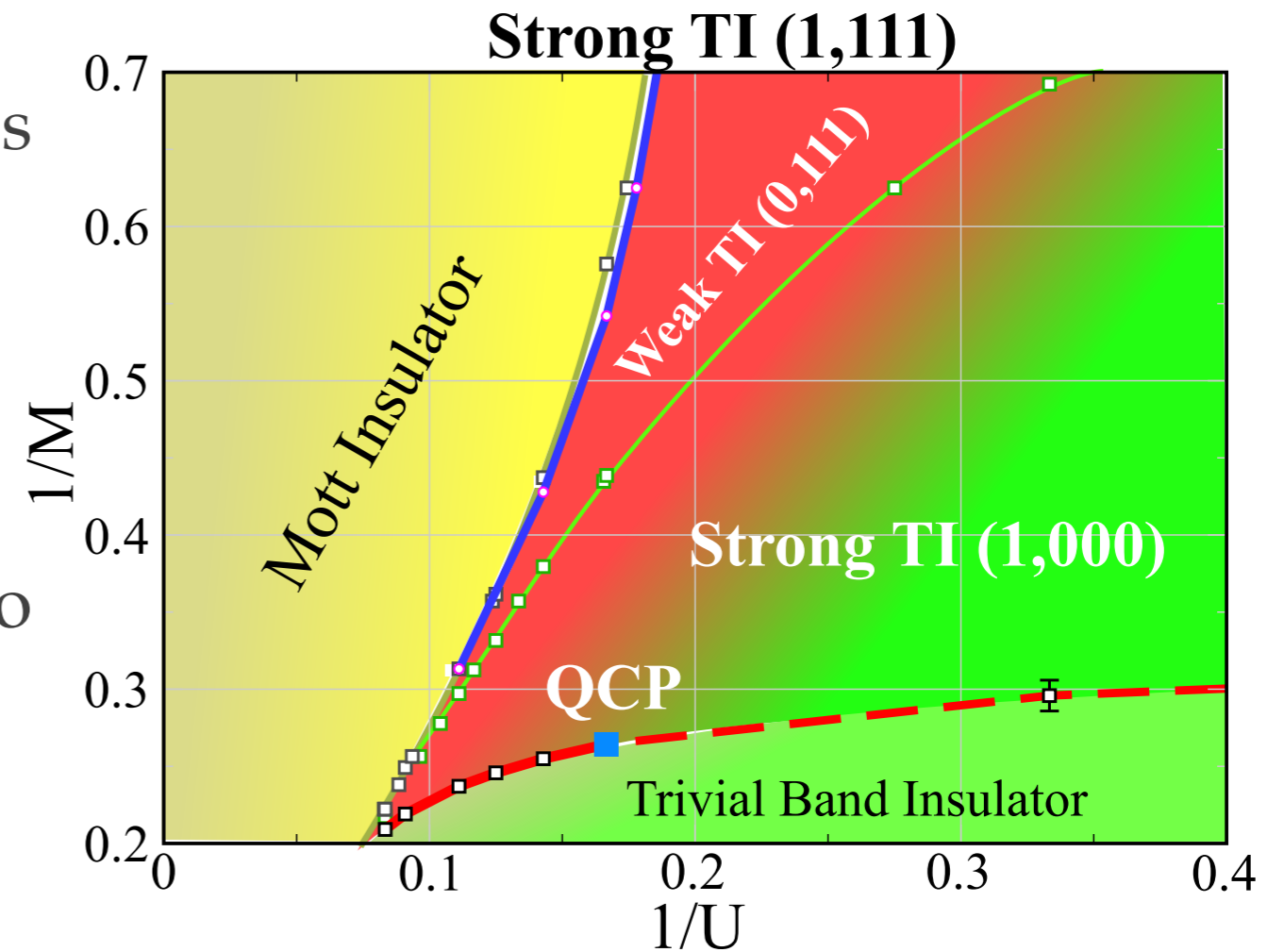


Generalisation to 3D

Quantum Critical Point dominates physics also in the 3D case.

1st-order transition to **Strong TI**

Anomalous STI (1,111) preludes to Mott transition.



$U > U_c$:

No gap-closing

No suppression of any symmetries

connected to the topological ordering.