From Number Theory to Dynamics of ac+dc driven Frenkel-Kontorova model

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1. Analogies with magnetic systems

- Axial NNN Ising model describes many interesting systems [1]

$$H = -\frac{J_0}{2}\sum_{i,j,j'} S_{i,j}S_{i,j'} - J_1\sum_{i,j} S_{i,j}S_{i+1,j} - J_2\sum_{i,j} S_{i,j}S_{i+2,j}$$

- At $T = 0$ only three possible spin configurations along z-axis form ground state (↑↑↑↑↑↑↑↑↑↑↑↑) for $p = -J_2/J_1 < 1/2$
(↑↑↑↑↑↑↓↓↓↓↓↓) for $p = -J_2/J_1 > 1/2$
(↑↑↓↓↑↑↓↓↑↑↓↓) for $p = -J_2/J_1 = 1/2$
where $J_0 > 0$, $J_1 > 0$ and $J_2 < 0$.

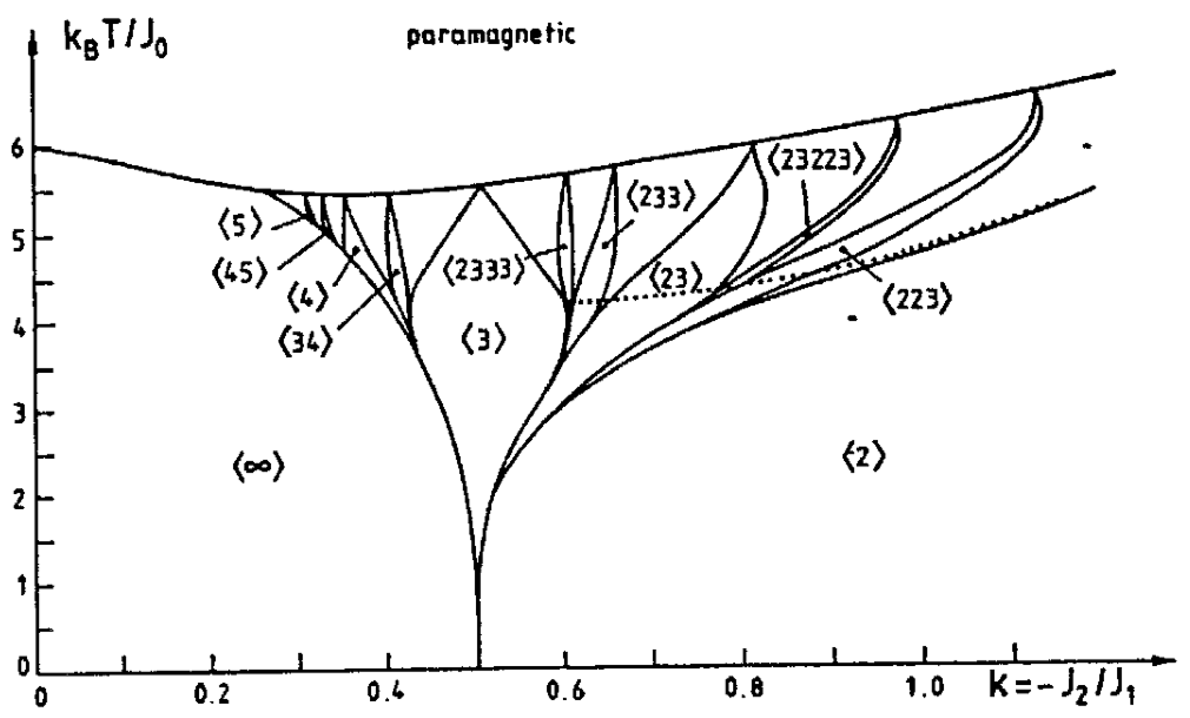

Fig. 1: Phase diagram of ANNNI Ising model with numerous comensurate phases taken from [2].

- Labels for different modulated structures at different temperatures
$\langle 2^2\rangle = \langle 22\rangle \rightarrow$ ... ↑↑↓↓↑↑↓↓↑↑↓↓↑↑↓↓↑↑↓↓↑↑↓↓ ...
$\langle 3^2\rangle = \langle 33\rangle \rightarrow$ ... ↑↑↑↓↓↓↑↑↑↓↓↓↑↑↑↓↓↓↑↑↑↓↓↓ ...
$\langle 2^2 3\rangle = \langle 223\rangle \rightarrow$ ... ↑↑↓↓↑↑↑↓↓↑↑↓↓↓↑↑↓↓↑↑↑ ...
$\langle\infty\rangle \rightarrow$ ... ↑↑↑↑↑↑↑↑↑↑↑↑ ...
- Phenomenological free energy

$$F = \sum_j \left[\Phi_1(M_j) + \Phi_2(M_j, M_{j+1}) + \Phi_3(M_j, M_{j+1}, M_{j+2}) + \cdots\right]$$

where

$$\Phi_1(u) = \frac{K}{(2\pi)^2}\left[1-\cos(2\pi u)\right] \qquad \Phi_2(u,v) = \frac{1}{2}\left[v-u-\mu\right]^2$$

2. Frenkel-Kontorova model

- Standard Frenkel-Kontorova model is a simple classical model which describes a chain of particles, usually identical, coupled to their nearest neighbours via potential $V_{\rm int}(u_l, u_{l+1}) = (u_{l+1}-u_l-\mu)^2$ and subject to the substrate potential (see Fig. 2 below)

$$V(u) = \frac{K}{(2\pi)^2}\left[1-\cos(2\pi u)\right]$$

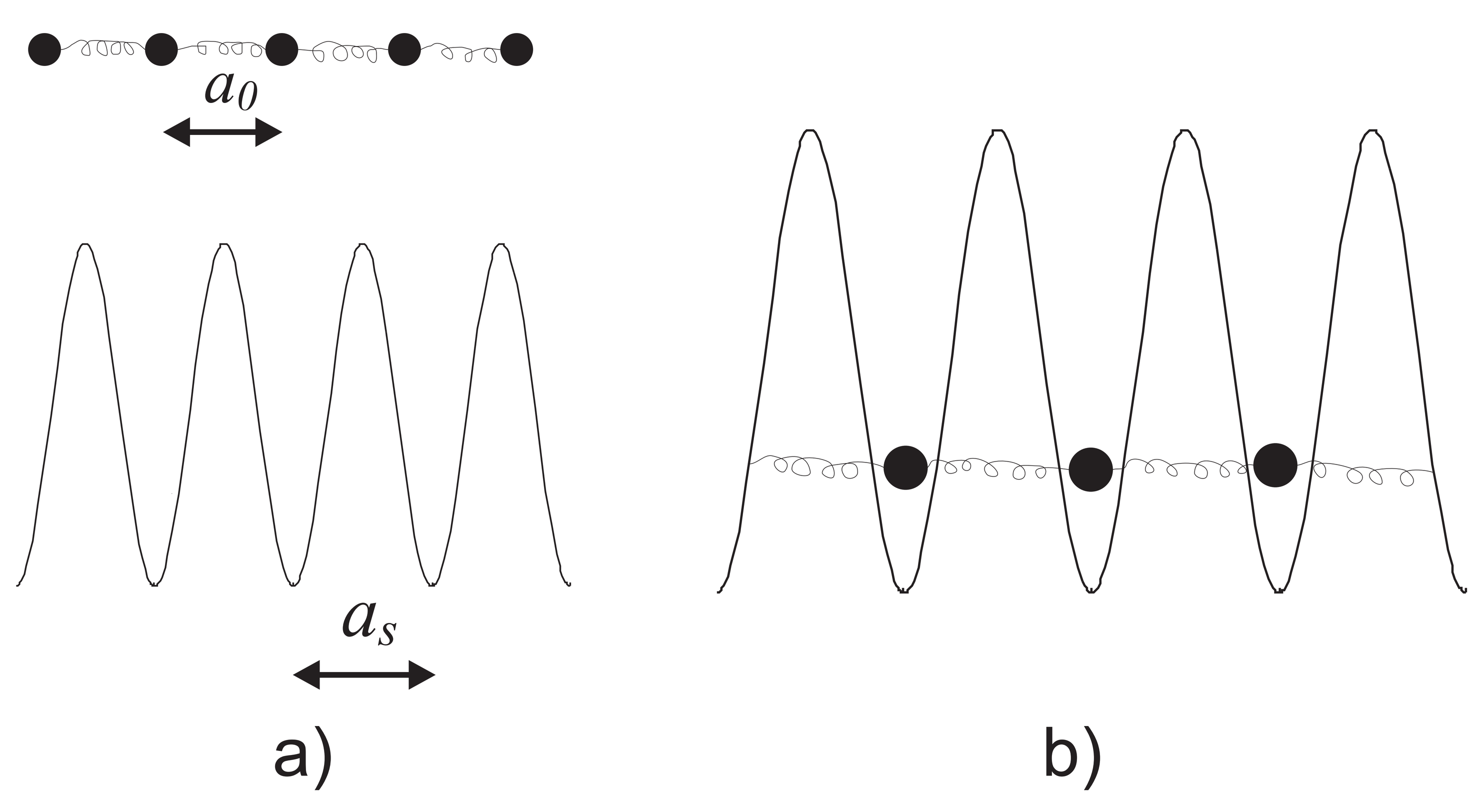

Fig. 2: a) Chain of particles and sinusoidal substrate potential and b) chain of particles subjected to substrate potential.

- Dissipative ac+dc driven overdamped Frenkel-Kontorova model

$$\dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u_l) + \bar{F} + F_{\rm ac}\cos(2\pi\nu_0 t)$$

- Generalized substrate potential – assymetric deformable potential (ASDP), see [3]

$$V(u) = \frac{K}{(2\pi)^2}\frac{(1-r^2)^2\left[1-\cos(2\pi u)\right]}{\left[1+r^2+2r\cos(\pi u)\right]^2}$$

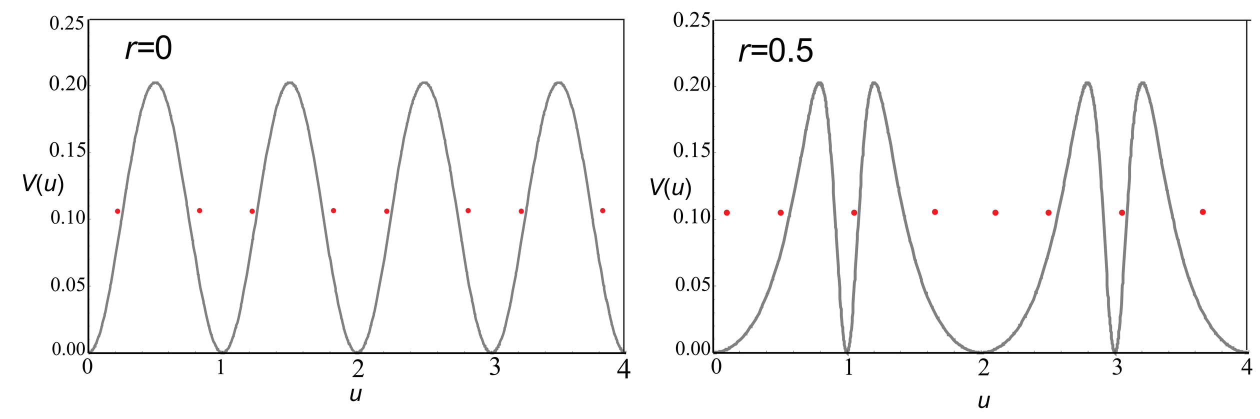

Fig. 3: Particles moving in asymmetric deformable potential for $\omega = \frac{1}{2}$, $K = 4$, and two different values of the shape parameters $r = 0$ and $r = 0.5$.

- A typical observable in such a non-linear classical model is the response function $\bar{v}$. One can also compute the largest Lyaponov exponent (See Fig. 4) [4,5].

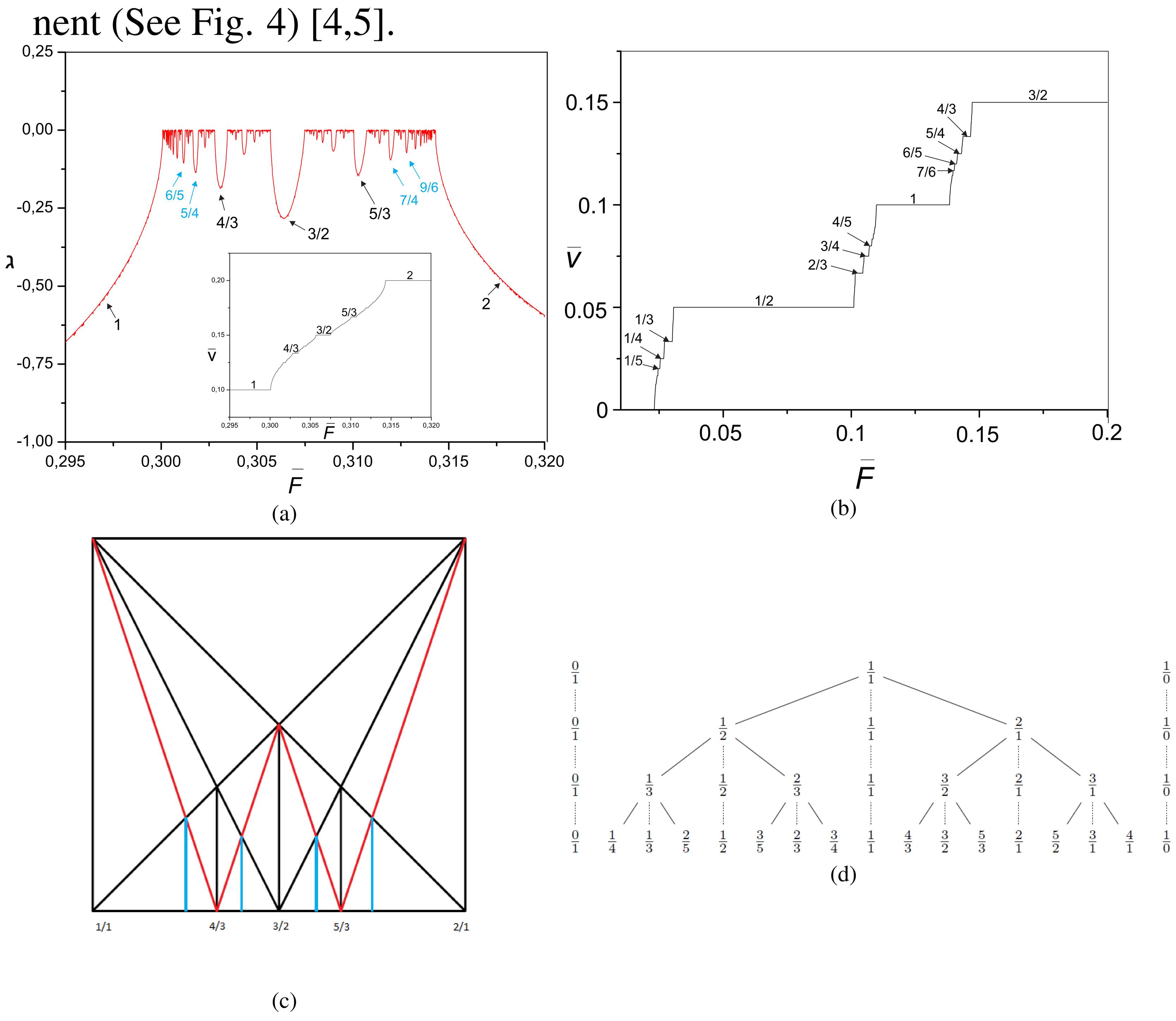

Figure 1: (a) Lyapunov exponent as a function of average driving force for $r = 0.01$ ($\omega = 1/2, K = 4, F_{ac} = 0.2, \nu_0 = 0.2$). The inset shows the response function $\bar{v}(\bar{F})$. (b) The average velocity as a function of average driving force for $F_{\rm ac}$ = 0.55, and r = 0.2, ω = 1/2, K = 4, ν_0 = 0.2. (c) Farey construction. (d) Stern -Brocot tree

3. Conclusion

- We computed the largest Lyaponov exponent for various ranges of parameters and chaotic behaviour was not observed.
- Competing interactions, within ac+dc driven FK model, lead to complex dynamics and apperence of Shapiro steps.
- The steps follow the Farey construction for certain ranges of parameters.

References

1. W. Selke, Phys. Repts. **170** 213, (1988)
2. L. M. Flora, J. J. Mazo, Adv. Phys. **45** 505, (1996)
3. B. Hu and J. Tekic, Phys. Rev. E **72**, 056602 (2005)
4. J. Odavić, P. Mali, J. Tekić, Phys. Rev. E **91**, 052904 (2015)
5. Symmetry transformation relation: $\bar{v} = \frac{i\omega + ja}{m}\nu_0$