

Relaxation dynamics of weakly interacting quantum systems

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Quench of an isolated quantum system

- ▶ Sudden change of a parameter

$$\text{Hamiltonian} = \begin{cases} H_0 & \text{for } t < 0 \\ H_0 + g H_1 & \text{for } t \geq 0 \end{cases}$$

- ▶ Time evolution of the state with $H = H_0 + g H_1$

$$|\psi(t=0)\rangle = |\psi_0\rangle$$

$$|\psi(t \geq 0)\rangle = \exp(-iHt)|\psi_0\rangle = \sum_n \exp(-iE_n t) |n\rangle \langle n|\psi_0\rangle$$

- ▶ Thermalization of **isolated** system?

$$\langle A \rangle_t = \langle \psi(t) | A | \psi(t) \rangle \xrightarrow{t \rightarrow \infty} \langle A \rangle_{\text{therm}} = \frac{\text{Tr} A e^{-\beta H}}{\text{Tr} e^{-\beta H}} \quad ?$$

with $\beta = 1/(k_B T)$ from $\langle H \rangle_{\text{therm}} = E = \langle \psi_0 | H | \psi_0 \rangle$

Statistical mechanics / Integrable systems

- ▶ Fundamental postulate of statistical mechanics:

All accessible microstates are equally probable

⇒ Gibbs ensemble $\rho_{\text{Gibbs}} \propto e^{-\beta H}$

- ▶ Integrable systems: No thermalization

Kinoshita et al, Nature **440** ('06)

Rigol et al, PRL **98** ('07)

Cazalilla, PRL **97** ('06)

Eckstein & Kollar, PRL **100** ('08)

...

- ▶ Many ($\propto L$) constants of motion I_α
- ▶ $[H, I_\alpha] = 0 \Rightarrow \langle I_\alpha \rangle_t = \langle I_\alpha \rangle_0 = \text{const}$
- ▶ Much fewer accessible states $\Rightarrow \rho_{\text{Gibbs}}$ not appropriate

- ▶ Nonthermal steady state: Statistical description

- ▶ Generalized Gibbs ensemble ρ_{GGE}

Rigol et al, PRB **74** ('06)

$$\rho_{\text{GGE}} \propto \exp\left(-\sum_{\alpha} \lambda_{\alpha} I_{\alpha}\right) \quad \text{with} \quad \langle I_{\alpha} \rangle_{\text{GGE}} = \langle I_{\alpha} \rangle_0$$

⇒ often correct, but not always

Gangardt & Pustilnik, PRA **77** ('08)

Barthel & Schollwöck, PRL **100** ('08)

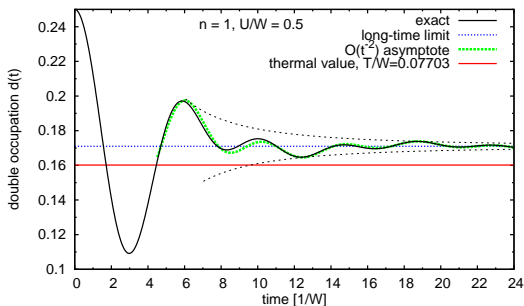
Kollar & Eckstein, PRA **78** ('08)

Relaxation in an integrable system

Integrable $1/r$ Hubbard chain: Quench from $U = 0$ to $U = 0.5$

Kollar & Eckstein, PRA **78** ('08)

$$d(t) = \langle n_{i\uparrow} n_{i\downarrow} \rangle_t$$



Nonthermal steady state: described by GGE

$$\lim_{t \rightarrow \infty} d(t) = d_{\text{GGE}} \neq d_{\text{therm}}$$

Thermalization in nonintegrable systems

▶ Nonintegrable systems:

Thermalization possible

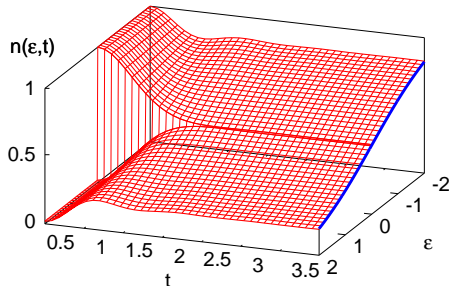
Kollath et al, PRL **98** ('07)
Rigol et al, Nature **452** ('08)
Barmettler et al, PRL **102** ('09); NJP **12** ('10)
Rigol, PRL **103** ('10)

▶ Fermionic Hubbard model:

$$H = \sum_{k\sigma} \epsilon_k a_{k\sigma}^\dagger a_{k\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \quad (\text{in } d=\infty / \text{noneq-DMFT})$$

Eckstein, Kollar, Werner, PRL **103** ('09); PRB **81** ('10)

Quench from $U=0$ to $U=3.3$:



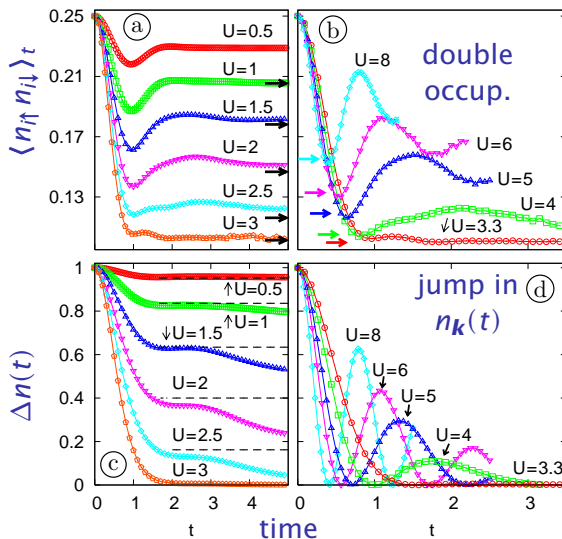
blue line: equilibrium
momentum distribution
for $U=3.3$ and $T_{\text{eff}}=0.84$

momentum distribution **thermalizes**

Thermalization in the Hubbard model

Interaction quench from 0 to U :

Eckstein, Kollar, Werner, PRL **103** ('09)



Vicinity of an integrable point: Prethermalization

Near integrable point:

▶ $H_0 = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$

⇒ integrable

▶ Quench to

$H_0 + g H_1$ with $|g| \ll 1$

⇒ long-lived state for

$$\frac{\text{const}}{g} \ll t \ll \frac{\text{const}}{g^2}$$

“Prethermalization”

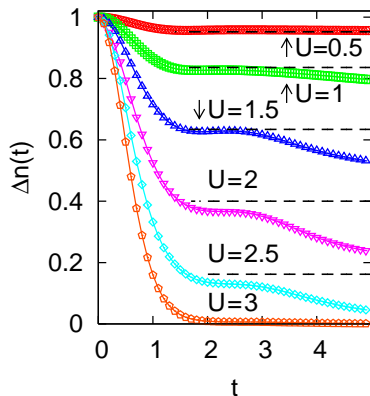
Berges et al, PRL **93** ('04)

Moeckel & Kehrein, PRL **100** ('08)

Moeckel & Kehrein, Ann. Phys. **324** ('09)

Interaction quench from 0 to U :

Fermi surface discontinuity



Eckstein, Kollar, Werner, PRL **103** ('09)
see also Uhrig, PRA **80** ('09)

Second order perturbation theory

The “Factor 2” Theorem:

Moeckel & Kehrein, Ann. Phys. **324** ('09)

- ▶ Quench from $H_0 = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$ to $H = H_0 + g H_1$
- ▶ Observable A with $[H_0, A] = 0$ and $\langle A \rangle_{t=0} = 0$
- ▶ Prethermalization plateau for $\frac{\text{const}}{g} \ll t \ll \frac{\text{const}}{g^2}$

$$\lim_{t \rightarrow \infty} \int_0^t \frac{dt'}{t} \langle A \rangle_{t'} = \underbrace{2 \langle \tilde{\psi}_0 | A | \tilde{\psi}_0 \rangle}_{=: A_{\text{pretherm}}} + O(g^3)$$

where $|\tilde{\psi}_0\rangle =$ ground state of H up to order g^2

Integrable vs. nonintegrable systems

Integrable systems

vs. Nearly integrable systems

(Q1) Relaxation to nonthermal (quasi-)steady state

Nonthermal steady state \longleftrightarrow Prethermalization plateau

(Q2) Statistical description of (quasi-)steady state

Generalized Gibbs ensemble

$$\rho_{\text{GGE}} \propto \exp\left(-\sum_{\alpha} \lambda_{\alpha} I_{\alpha}\right) \longleftrightarrow \rho_{\widetilde{\text{GGE}}} \propto \exp\left(-\sum_{\alpha} \lambda_{\alpha} \tilde{I}_{\alpha}\right)$$

GGE with exact
constants of motion I_{α}

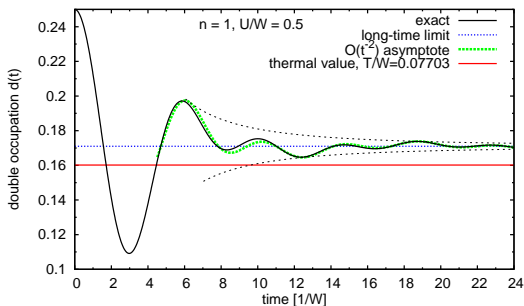
GGE with approximate
constants of motion \tilde{I}_{α} ?

Q1: Nonthermal vs. prethermalized

Integrable $1/r$ Hubbard chain: Quench from 0 to U

Kollar & Eckstein, PRA **78** ('08)

$$d(t) = \langle n_{i\uparrow} n_{i\downarrow} \rangle_t$$



$$\lim_{t \rightarrow \infty} d(t) = \frac{n^2}{4} + \frac{n^2(2n-3)}{6}U + O(U^2) = d_{\text{GGE}} \neq d_{\text{therm}}$$

Prediction for prethermalization plateau:

$$d_{\text{pretherm}} = \frac{n^2}{4} + \frac{n^2(2n-3)}{6}U + O(U^2) \quad !!$$

Q2: Approximate constants of motion

Before quench:

- ▶ $H_0 = \sum_{\alpha} \epsilon_{\alpha} I_{\alpha}$ with $I_{\alpha} = a_{\alpha}^{\dagger} a_{\alpha}$ (bosons or fermions)
- ▶ $[I_{\alpha}, I_{\beta}] = 0$, $[H_0, I_{\alpha}] = 0$
- ▶ Basis: $I_{\alpha} |\mathbf{n}\rangle = n_{\alpha} |\mathbf{n}\rangle$
- ▶ System in ground state $|\psi_0\rangle$ of H_0

After quench:

- ▶ $H = H_0 + g H_1$ with $[H_0, H_1] \neq 0$ and $|g| \ll 1$
- ▶ H_1 can be expressed with a_{α}^{\dagger} and a_{α}
- ▶ $|\tilde{\psi}_0\rangle =$ ground state of H

Construction of approximate constants of motion

- ▶ Canonical trafo: $S = gS_1 + \frac{1}{2}g^2S_2 + O(g^3)$

Harris & Lange, PR **157** (1963)

$$e^S H e^{-S} = H_0 + g(H_1 + [S_1, H_0]) \\ + g^2\left(\frac{1}{2}[S_2, H_0] + [S_1, H_1] + \frac{1}{2}[S_1, [S_1, H_0]]\right) + O(g^3)$$

- ▶ With $\tilde{I}_\alpha = e^{-S} I_\alpha e^S$ and $|\tilde{n}\rangle = e^{-S} |n\rangle$:

$$H = \sum_{\alpha} \epsilon_{\alpha} \underbrace{\tilde{I}_{\alpha}}_{\tilde{n}} + \sum_{\tilde{n}} |\tilde{n}\rangle \langle \tilde{n}| (E_{\tilde{n}}^{(1)} + E_{\tilde{n}}^{(2)}) + O(g^3) \\ = \text{approx. const. of motion of } H$$

- ▶ Generalized Gibbs ensemble:

$$\rho_{\text{GGE}} = \exp\left(-\sum_{\alpha} \lambda_{\alpha} \tilde{I}_{\alpha}\right) \quad \text{with} \quad \langle \tilde{I}_{\alpha} \rangle_{\text{GGE}} \stackrel{!}{=} \langle \tilde{I}_{\alpha} \rangle_0$$

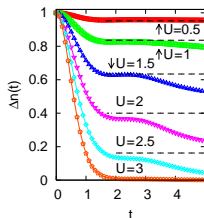
Statistical prediction for pretherm. plateau

GGE prediction $\langle A \rangle_{\widetilde{\text{GGE}}}$ vs. pretherm. plateau $\langle A \rangle_{\text{pretherm}}$:

Kollar, Wolf, Eckstein, PRB **84** ('11)

- ▶ Simplest observable: $A = I_\alpha = a_\alpha^\dagger a_\alpha$

$$\Rightarrow \langle I_\alpha \rangle_{\widetilde{\text{GGE}}} = \langle I_\alpha \rangle_{\text{pretherm}} + O(g^3) \quad \mathbf{!!}$$



- ▶ General observable: $A = I_{\alpha_1} \cdot \dots \cdot I_{\alpha_m}$

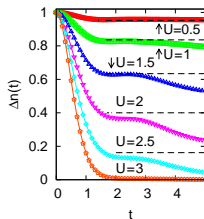
$$\Rightarrow \langle A \rangle_{\widetilde{\text{GGE}}} - \langle A \rangle_{\text{pretherm}}$$

$$= \langle I_{\alpha_1} \rangle_{\tilde{\psi}_0} \cdot \dots \cdot \langle I_{\alpha_m} \rangle_{\tilde{\psi}_0} - \langle I_{\alpha_1} \cdot \dots \cdot I_{\alpha_m} \rangle_{\tilde{\psi}_0} + O(g^3)$$

similar to GGE criteria in Kollar & Eckstein, PRA **78** ('08)

Conclusion

- ▶ 'Dressed' GGEs predict prethermalization in nearly integrable systems (e.g., for $n_{k\sigma}$ and d in small- U quenches)



- ▶ Integrable / nearly integrable systems are connected, and described by one statistical theory
- ▶ Nonthermal steady states in integrable systems evolve into prethermalization plateaus away from integrability
- ▶ Statistical mechanics works, if done right